Institution: The University of Edinburgh/Heriot-Watt University (Maxwell Institute)

Unit of Assessment: B10, Mathematical Sciences

a. Overview

The Maxwell Institute (MI) was established in 2005 by the University of Edinburgh (UoE) and Heriot-Watt University (HWU) to create a world-leading research centre that spans the full spectrum of mathematical sciences and impacts on the many disciplines underpinned by mathematics. The MI unites the research activities in mathematical sciences of UoE's School of Mathematics and HWU's School of Mathematical and Computer Sciences. It exploits the complementary expertise in both institutions and advances their strong track record of collaboration, best exemplified by the establishment in 1990 and management of the International Centre for Mathematical Sciences (ICMS). It forms part of a broader consortium, the Edinburgh Research Partnership in Engineering and Mathematics (ERPem), which combines the research capacity of UoE and HWU in both engineering and mathematics and promotes work at the interface between the two disciplines.

Key and distinctive strengths of the MI are its extensive coverage of mathematical sciences and the dynamism of its interactions with disciplines beyond mathematics. This submission presents the activities of our groups in the following areas: Algebra, Geometry & Topology; Analysis & PDEs; Applied & Computational Mathematics; Mathematical Physics; Operational Research; Probability & Statistics. The boundaries between these groups are permeable and many staff are active in more than one group. The MI's research contribution extends beyond the scope of UoA10 and MI staff are included in the REF submissions of HWU and UoE in UoA 19 (Business & Management Studies) and UoA 20 (Law).

b. Research strategy

Our vision for the MI is that of a world-leading research centre in the mathematical sciences, distinguished by vigorous interactions between mathematical research themes and strong connections to other disciplines. To deliver this vision, we have adopted a coordinated strategy pursuing the following objectives:

- to conduct research at the highest international level and support our prominent research leaders;
- to develop key areas of excellence and stimulate work at the interface between these;
- to invest in the most promising early-career researchers and provide them with the necessary support to realise their potential as leaders;
- to increase the reach and impact of our research by promoting collaborations with other academic disciplines and industry, and through public engagement;
- to diversify our grant portfolio by winning funding from a broader range of public and private sponsors;
- to support ICMS vigorously by investing staff time for management and fundraising efforts as well as workshop organisation.

This strategy has achieved major successes since 2008:

- our research leaders have been awarded 8 major ERC and EPSRC fellowships worth more than £5.5M;
- our investment in promising early-career researchers has led to 6 first grants awarded by EPSRC in the last three years;
- our body of interdisciplinary work has grown thanks to research grants worth £3M in areas such as the digital economy, energy, computational chemistry, climate, and developmental and evolutionary biology;

we have successfully diversified our funding base with more than £6.6M secured from sponsors other than EPSRC, including four other UK research councils (BBSRC, ESRC, NERC, STFC), the EU, charities and industrial sources.

The critical mass achieved in analysis and PDEs (16 staff), in computational mathematics (17 staff) and in mathematical physics (15 staff) has been decisive in attracting more than £8.5M in major grants that boost the MI's infrastructure and its capacity to deliver world-leading research:

the **Centre for Analysis and Nonlinear Partial Differential Equations** (CANPDE), funded by a 2007 EPSRC S&I award and by the Scottish Funding Council (SFC) for £3.4M, has cemented the MI's position as a leading research centre in analysis;

the £4.5M S&I award (2009) funding the centre for **Numerical Algorithms and Intelligent Software** (NAIS) has intensified our activities in computational mathematics and established new, vital links with computer scientists and industrial users;

new funding from STFC worth £1M (2011) has supported the **Tait Institute** which provides a focal point for mathematical physics in the MI and UoE's School of Physics.

Our increased emphasis on industrial engagement and knowledge transfer has led to the establishment in 2010 of the **Scottish Financial Risk Academy** (SFRA) which provides a conduit for our research in financial and actuarial mathematics to impact on the financial industry, and by the formation in 2012 of the **Actuarial Research Centre**, a joint venture with the Institute and Faculty of Actuaries whose benefits include industry-sponsored PhD studentships. Our position as an international hub for mathematical sciences has been strengthened by £3M awarded to the MI to support the **International Centre for Mathematical Sciences** (ICMS) in its successful programme of workshops (over 70 since 2008) and enable its move to new premises in 2010.

Our strategy has been developed in discussion with the MI's Advisory Committee (J Ball, Oxford; J Elgin, Imperial; J Pipher, Brown; D Arnold, Minnesota; and P Embrechts, ETH Zurich) who steered the MI during its establishment phase (2005-2010). Its implementation by the MI co-directors (currently Leimkuhler and Szabo) and research-group leaders has been facilitated by an increased integration of the mathematical activities at UoE and HW, by joint appointment committees, and by the infrastructure grants awarded to the MI. We have also taken full advantage of institutional initiatives for increasing research intensity through staff recruitment, for example HWU's Global Platform and UoE's Chancellor's Fellowship schemes which have funded 10 new appointments. The strategy has built large, coherent research groups. We summarise below the research interests and activities of each group, emphasising how these align with our strategy and highlighting the most significant achievements of the 2008-13 period. The total number of staff (including research assistants) currently in each group and the names of the group leaders are indicated in brackets.

Algebra, Geometry & Topology (26 staff; lead**:** Gordon/Howie). The group's expansion since RAE2008 has followed a double strategy: existing strengths in noncommutative algebra and representation theory have been built up with the recruitment of Jordan, Sierra and Wemyss, and research in algebraic geometry has expanded with the recruitment of Bayer, Hering, Martens and Pridham. The group has enjoyed remarkable success by securing 6 major 5-year fellowships since 2008: Bayer (ERC), Gordon (EPSRC), Pridham (EPSRC), Smoktunowicz (EPSRC and ERC), Wemyss (EPSRC). Combined with other successes, grant income has exceeded £6.6M. Research in the group now spans algebraic geometry, categories and their applications, geometric group theory, higher dimensional algebraic topology and knot theory, Lie theoretic representation theory, noncommutative algebra and geometry, symplectic geometry, and semigroup theory. Recent research highlights include: Bayer's systematic explanation of the relation between wall-crossing for Bridgeland stability and the minimal model program; Martens construction of wonderful

compactifications of arbitrary reductive groups; Pridham's work establishing the foundation of Simpson's n-stacks in the context of derived algebraic geometry; Leinster's new categorical notion of magnitude with application throughout mathematics and to ecology; Howie's classification and decidability theorems for finitely presentable residually free groups; Smoktunowicz's definitive answers to several questions of Zelmanov; and Gordon's discovery of the first representation-theoretic proof of Macdonald positivity.

The achievements of the group have been recognized by a European Mathematical Society Prize (Smoktunowicz); invited lectures at the International Congress of Mathematicians (Gordon) and at the European Congress of Mathematicians (Smoktunowicz); and 20 or so visiting positions at leading institutions worldwide, including visiting professorships at Göttingen, ETH and MSRI. Since 2008, 5 early-career researchers (ECRs) from the group have taken up permanent academic posts. The group co-organised many high-profile international conferences including Atiyah80, the first European-Nordic Congress of Mathematicians, and 7 workshops at research centres such as CIRM Luminy, ICMS and MSRI. It developed and helps to lead the highly successful Anglo-French-German Representation Theory Network funded by CNRS, DFG and EPSRC; nationally it co-founded the seminar series ARTIN (algebra and representation theory), GLEN (algebraic geometry), North British Geometric Group Theory Seminar, and Scottish Topology.

Analysis & PDEs (16 staff; lead: Carbery/Rynne). The award of the £3.4M EPSRC and SFC grants that founded CANPDE at the end of 2007 testifies to the strength and international reputation of the group, which enjoys EPSRC Major Player status. The group has recently been considerably strengthened with the appointments of 5 leading researchers: Beck, Blue, Dreher, Karakhanyan and Oh. It has initiated and successfully developed the many CANPDE activities: workshops, mini-symposia, seminars, two international conferences, and the highly innovative series of 'crash courses' co-organised with the Oxford Centre for Nonlinear PDEs. The group is a founding member of the LMS Harmonic Analysis and PDE Research Network. It has hosted 15 postdoctoral fellows and RAs since 2008, and its graduating PhD students regularly obtain prestigious postdoctoral positions. Blue and Karakhanyan have both won EPSRC first grants.

The group's research is mainly concentrated around PDEs, harmonic analysis, spectral theory and ODEs. Research on nonlinear differential equations covers dissipative equations (Beck), dispersive, hyperbolic and kinetic PDEs (Blue, Bournaveas, Oh), and free-boundary problems in various continuum theories (Choffrut, Coutand, Karakhanyan). Group members apply ideas from harmonic analysis to number theory, combinatorics, discrete and differential geometry, and geometric measure theory (Carbery, Wright), to elliptic and parabolic PDEs (Dindos), and to nonlinear hyperbolic and dispersive PDEs (Blue, Bournaveas, Oh). Boulton and Rynne study the spectral theory of linear and nonlinear problems and their applications in mathematical physics. Gyongy analyses finite difference schemes for a wide variety of differential equations.

Recent research highlights include: the first treatment of nonlinear stability for time-periodic viscous shock solutions with marginally stable spectrum (Beck); a sharp geometric condition for the existence and smoothness of weak solutions to the fully nonlinear Monge-Ampère equation governing reflector systems (Karakhanyan); the discovery and exploitation of hidden symmetries leading to new decay estimates for the wave equation on the Kerr spacetime (Blue); the establishment of a local one-to-one correspondence between steady states and co-adjoint orbits in the incompressible Euler equations (Choffrut); the almost-sure well-posedness of nonlinear evolution equations below critical regularity (Oh); the complete resolution of the 35-year-old conjectures of Igusa on polynomial congruences in the two variable case, obtaining the best possible estimates (Wright); the first proof of the multilinear Kakeya theorem avoiding the general machinery of algebraic topology (Carbery); the discovery of a counterexample to a long-standing conjecture regarding variational eigenvalues of nonlinear PDEs (Rynne).

Applied & Computational Mathematics (29 staff; lead: Leimkuhler/Lord). A visible success of the MI is the expansion of the Applied and Computational Mathematics group, awarded more than £6M in grants since 2008. The establishment of the NAIS centre in 2009 has greatly enhanced its position in computational mathematics, helping to attract excellent recruits (Banjai, Branicki, Gimperlein, Goddard, Lindsay, Loisel, Maddison and Schmuck) and boosting interactions with the broader high performance computing community. The group has also become a leader in application-oriented stochastic modelling (ranging from probabilistic game theory to numerical SDEs and SPDEs), furthering its links with Probability & Statistics. Additional strengths include mathematical biology, fluid dynamics, applied dynamical systems, and PDE-based modelling.

Research on numerical methods for differential equations spans ODEs, PDEs (Banjai, Duncan, Gimperlein, Gyongy, Leimkuhler, Lindsay, Lord), SDEs and SPDEs (Gyongy, Lord, Malham), with applications that include fluids, phase-field models, electromagnetics and porous media. Grants from NSF, EPSRC and ERC have supported the group's growing activities in molecular dynamics (Goddard, Leimkuhler). Research in mathematical biology has expanded from the areas of tumour-growth and cell modelling and ecology (McDougall, Sherratt, Painter, White) to include genetic models, evolutionary biology (Antal) and neurosciences (Lord, Popovic). Research in asymptotics and dynamical systems (Olde Daalhuis, Popovic, Beck, Coutand) establishes strong connections with Analysis & PDEs. Collaborations with engineering, particularly petroleum engineering (Lacey, Lord, Duncan), have benefitted from the EPSRC-funded Bridging-the-Gaps initiative and the MI's membership of the ERPem consortium. The group's activities in geophysical fluid dynamics, supported by NERC, have been recognised by the award of the 2010 Adams Prize to Vanneste. They have been strengthened by the recruitment of Branicki, Maddison and Chumakova and will benefit from the MI's partnership in the newly funded NERC Doctoral Training Partnership Edinburgh Earth and Environment (E3 DTP).

Research highlights include: Antal's widely reported papers on pancreatic cancer and evolutionary aspects of cancer; Branicki's demonstration of fundamental limitations of the polynomial (Wiener) chaos expansions widely used in uncertainty quantification; Leimkuhler's discovery of an explicit non-Markovian integration method for Brownian dynamics with unexpected second-order accuracy for the invariant measure; Lord's work on error bounds for SPDE numerical schemes; Painter's fundamental work on cancer and chemotaxis modelling; Sherratt's series of papers providing the first analytical study of banded vegetation patterns in semi-arid environments; Vanneste's elucidation of the connection between the asymptotics of slow manifolds and spontaneous wave generation in the atmosphere and ocean.

Mathematical Physics (15 staff; lead: Figueroa-O'Farrill/Szabo). The Edinburgh Mathematical Physics Group (EMPG) has strengthened since 2008 with the appointment of 4 further academic staff: Dolan, Doiku, Lucietti and Sämann, the latter two EPSRC Career Acceleration Fellows. It benefits from the presence of Sir Michael Atiyah, an honorary professor at UoE. Since 2008, EMPG has hosted 5 Postdoctoral Fellows and PDRAs, graduated 8 PhDs, published 128 research articles and attracted a total grant income of £1.9M. This includes funding from STFC in the form of a two-year responsive mode grant in 2009, and a three-year consolidated grant in 2011 enabling the establishment of the Tait Institute and renewed for 3 years from 2014. The EMPG has gained a strong international visibility through its organisation of workshops and meetings, including a satellite meeting of the British Mathematical Colloquium in 2010 on string theory, and the Atiyah80 conference in 2009. EMPG was also the grant holder of the LMS-funded North British Mathematical Physics Seminars and is a node of the Classical and Quantum Integrability Network. Group members are sought-after speakers and organisers for international conferences, schools, colloquia and seminars. They participate in and organise scientific programmes at high-profile institutes worldwide including the Perimeter Institute, the Kavli Institute, the Schrödinger Institute,

and the Simons Center.

Research in the group spans gravitational physics and AdS/CFT, conformal field theory, solitons and monopoles, integrable systems and statistical mechanics. There are many common interests with the Algebra, Geometry & Topology group: EMPG has earned an international reputation for its achievements in noncommutative geometry, K-theory, algebraic geometry, and higher algebraic and geometric structures arising in string theory and M-theory. Research highlights include: a novel four-dimensional version of the Skyrme model based on Donaldson theory (Atiyah & Schroers); the development of constructive algebro-geometric techniques for integrable systems (Braden); a classification of near-horizon geometries of 5-dimensional extremal vacuum black holes (Lucietti); a proof of the homogeneity conjecture for supergravity backgrounds (Figueroa-O'Farrill); a construction and microscopic description of the most general extremal non-BPS black hole in supergravity models (Simón); the discovery and twistor description of the field equations for a non-abelian (2,0) superconformal tensor multiplet in 6 dimensions (Sämann); and the construction of higher quantization schemes for string backgrounds (Szabo).

Operational Research (12 staff; lead: Gondzio). Since 2008 the group has been strengthened by the recruitment of Richtárik, Buke, Garcia and Pearson and the presence of a strong team of PDRAs supported by more than £1M of EPSRC investment in the Digital Economy and Energy. Research focuses on mathematical optimization and covers all aspects of modern optimization algorithms: theoretical foundations, design and implementation, and applications to real-life problems. The group has broad expertise in several areas: interior point methods, simplex method, nonlinear programming, stochastic programming, combinatorial optimization and first-order methods. Recent research highlights include establishing the worst-case complexity bounds for interior point methods using inexact Newton methods and developing matrix-free variants of these methods (Gondzio); major contributions to the theory of coordinate descent methods now regarded as leading algorithms in big-data optimization (Richtárik). Implementation efforts have focused on the exploitation of matrix sparsity and the use of modern computer architectures (parallel, multicore and GPUs). The methods and software developed have been exploited in ongoing collaborations with industrial partners funding the group: SAS, Yahoo, Orange-FT, Arup, Statoil, Format International, Selex Galileo, EADS-Paris.

The group maintains a high international visibility through participation in scientific committees of high profile international conferences, plenary talks at major conferences (EUROPT, SPXIII), and the organisation of successful workshops (Advances in Large-Scale Optimization, 2012, Optimization in Big Data, 2013). The group is particularly well known for its contributions to huge-scale optimization (Gondzio, Grothey, Richtárik) and is perfectly placed to respond to the computational challenges resulting from the explosion of big data. In 2013 Richtárik was a long-term visitor of 'Theoretical Foundations of Big Data Analysis' at the Simons Institute in Berkeley. From 2014 the group will benefit from its participation in a new EPSRC CDT on Data Science.

Probability & Statistics (18 staff; lead: Foss/Gyongy). Successful strategic developments for the group since 2008 include its expansion into new application areas (energy, climate change and the built environment) and its enhancement of capacity in established areas (finance, communication networks, mortality, medicine and epidemiology, and forensic science). This growth is facilitated by 5 recent appointments (Shneer, Cruise, Rasonyi, Szpruch, Daly) who bring expertise across the areas of applied probability, operational research and stochastic analysis which underpin our theoretical and methodological research. Research highlights include: Rasonyi's characterisation of optimal investments in incomplete markets; important advances on numerical approximation of SDEs and SPDEs (Gyongy, Wiese, Szpruch, Malham); the use of data-reduction and model emulation techniques in the simulation of complex building models (Gibson); new asymptotic

results on Bayesian posteriors in high-dimensional, non-regular settings (Bochkina); extensive asymptotic results on random sums of heavy-tailed distributions and on random directed graphs (Foss, Zachary). The group benefits from the co-location of Biomathematics & Statistics Scotland (BIOSS), who participate in the MI seminar series in Statistics and in joint PhD supervisions. The group will contribute to both the NERC E3 DTP and the EPSRC Data Science CDT.

The success of the group since RAE2008 is evidenced by the award of a total of £800k from EPSRC for research on energy and climate change, and two best paper awards at the 2011 World Congress in Renewable Energy (Gibson). Interactions with users have been greatly enhanced by funding awards that support knowledge transfer in renewable energy and by the establishment of the Scottish Financial Risk Academy (SFRA). Principal collaborators include the universities of Cambridge, Oxford, ENS/INRIA, EURANDOM, Stanford, Novosibirsk, Humboldt, Lausanne) and many industrial partners (National Grid Plc., Scottish and Southern Energy, and major companies from the financial sector through the SFRA, see section d).

Future
The fulfilment of the main ambitions outlined in RAE2008 and the successes achieved since have put the MI in a strong position for the coming years. Key elements for its sustainability as a world-leading research centre are in place: international visibility; the continuing commitment of UoE and HWU to support the MI's ambitions in staff recruitment; a cohesive organisational structure; a high-quality staff that includes prominent research leaders and a large proportion (20%) of ECRs; a buoyant graduate school; and strong links beyond the mathematical sciences. Our strategy will exploit these strengths to ensure that the MI takes full advantage of the new opportunities that arise. Specifically, we aim:

- to target infrastructure funding and other large-scale grant opportunities (such as EPSRC programme grants), drawing on our successful S&I experiences;
- to stimulate applications for fellowships by staff at all career stages;
- to strengthen our potential for impact by broadening our links with other disciplines and with industry and by continuing to encourage multidisciplinary work;
- to expand graduate training in volume and in quality, responding to opportunities such as CDTs both in the mathematical sciences and in collaboration with other disciplines;
- to develop our research capacity at the interface between mathematical areas particularly algebra/geometry/mathematical physics, analysis/computational and probability/applied.

These measures will ensure that our research excellence extends to new emerging areas and yields increasing benefits.

c. People

Staffing strategy
The MI's staffing policy is to recruit outstanding mathematicians from across the world, and to offer its staff a highly supportive research environment and an attractive career path. Investment by the universities has enabled us to strengthen our staff; recruitment of early-career staff of exceptional research potential has been the priority. Since RAE2008, 35 independent academic researchers have been appointed (see group descriptions in b), many from top overseas institutions such as Harvard, Princeton, Albert Einstein Institute, EPFL and ETH. The MI now benefits from a well-balanced age structure and a rich mix of nationalities, with 35% academic staff below the age of 40 and a 45-35-20% split for staff originating from the UK, EU and rest of the world.

Fellowships. The success of our strategy is indicated by the prestigious research fellowships awarded to our staff. Fellowship holders during 2008-13 are: Bayer (ERC Starting Grant 2013-18), Beck (NSF Postdoctoral Research Fellowship 2006-9 and Sloan Research Fellowship 2012-13), Chlouveraki (EPSRC Postdoctoral, 2009-2012), Chumakova (NSF Postdoctoral Research, 2009-

12), Figueroa-O'Farrill (Leverhulme, 2009), Gordon (EPSRC Leadership, 2008-14), Hering (Oberwolfach Leibniz Fellowship, 2009-10), Leinster (EPSRC Advanced Research, 2006-11), Lucietti (EPSRC Career Acceleration, 2010-2014), Mackay (Royal Academy of Engineering, 2010), Pouchin (EPSRC Postdoctoral 2011-14), Painter (Leverhulme, 2011-13), Pridham (EPSRC Career Acceleration, 2011-16), Sierra (NSF Postdoctoral Research, 2008-2011), Sämann (EPSRC Career Acceleration, 2009-14), Sherratt (Leverhulme, 2009-11), Smoktunowicz (ERC Advanced Investigator, 2013-18, and EPSRC Advanced Research, 2006-11), Vanneste (Leverhulme, 2009-10), White (Royal Society of Edinburgh, 2010), Wemyss (EPSRC Early Career, 2013-18).

We recognise the immense value of postdoctoral fellowships for enabling young mathematicians to pursue independent, original work. We have invested accordingly, establishing internationally competitive fellowship schemes that attract outstanding ECRs to the MI. These schemes are:
Chancellor's Fellowships funding 5-year tenure-track posts leading to permanent lectureships or readerships and awarded to Jordan, Leinster, Martens, Oh and Pridham.
Seggie Brown Fellowships, funded by an endowment that supports, at any given time, 2 to 3 postdocs (9 since 2008).
Whittaker Fellowships currently supporting 5 independent ECRs in two-year positions.
In addition, the MI hosts many PDRAs funded by research grants (65 since 2008). As a result, we benefit from a vibrant community of ECRs who play a central role in seminar programmes, deliver research-level mini-courses and lead working groups on particular topics.

Visitors. Our environment is enhanced by the many mathematicians who visit Edinburgh to collaborate with MI staff and to engage with our diverse programme of activities which includes around 15 concurrent series of weekly seminars and many workshops (run by ICMS, CANPDE, NAIS and others). Extended visits (>10 days) are frequent – around 35 per year. Notable visitors since 2008 include Alexakis (Toronto), Asmussen (Aarhus), Cherednik (Chapel Hill), Dafermos (Princeton), Friedan (Rutgers), Junge (Champaign-Urbana), Kapouleas (Brown), Klainerman (Princeton), Kloeden (Goethe U), Koch (MPI Bonn), Krylov (Minnesota), Mariño (Geneva), Mazzeo (Princeton), Nesterov (Louvain), Nuñez (Swansea), Pego (Carnegie Mellon), Pipher (Brown), Pisier (Texas A&M), Saint-Raymond (Ecole Normale Supérieure), Schlag (Chicago), Solymosi (UBC), Tao (UCLA), Tadmor (Maryland), Tolsa (Barcelona), Zinn-Justin (Paris).

Several MI initiatives have attracted high-profile figures to Edinburgh in 2008-13. The *MI Colloquia*, a series of distinguished lectures accompanied by specialised workshops open to the entire UK community, invited Costello (Notre Dame), Pohorille (NASA), MacDuff (Stony Brook/Barnard), Young (Scripps), Murray (Oxford and Washington), Temme (Amsterdam), Arnold (Minnesota), Cucker (Hong Kong), Serre (Collège de France), Glynn (Stanford), Sturmfels (Berkeley) and Saunders (Stanford). The *MI Lectures* were given by Gowers (Cambridge), Zagier (MPI, Bonn) and Papanicolaou (Stanford), the *Whittaker Lectures* by Kronheimer (Harvard), Zeeman (Warwick), Melrose (MIT), Carlsson (Stanford), Stillwell (San Francisco) and Szemerédi (Rutgers). These events complement our seminar series, regular colloquia and meetings of the Edinburgh Mathematical Society.

Staff development
ECRs. New academic staff are assigned a mentor who provides advice on research goals, monitors progress towards these, and represents their interests. They receive light teaching and administrative loads and are offered flexible start-up packages to accelerate their career development. They are encouraged to participate in the organisation of research events such as ICMS workshops and to make appropriate grant applications. Applications to EPSRC's first grants are supported by direct investment by the MI which provides matching funds to extend the period of RA funding to 2 years or to fund PhD studentships. As a result 6 staff have been awarded first

grants. Fellows and RAs are also supported by mentors. They benefit from the implementation of the Concordat to Support the Career Development of Researchers (for which UoE and HW were awarded HR Excellence in Research Award by the EC in 2010) including opportunities to undertake modest teaching activities in small- and large-class settings and full access to the diverse development activities organised by the universities. Through membership of ERPem, funds are available for ECRs to participate in international research exchanges or to develop international funding bids; since 2012, 5 MI staff have taken advantage of this opportunity.

All staff benefit from annual reviews offering feedback on all aspects of their activities from the relevant senior staff. They attend a range of courses on teaching skills, supervision of research students and research staff, management and leadership. Our staff policy is geared towards research excellence: teaching and other duties are allocated flexibly to enable staff to make extended research visits or attend conferences; all staff have access to departmental travel funds. Sabbatical leave is granted on a regular basis (32 since 2008); additional leaves are systematically granted to staff securing funding for teaching replacement, with 6 fellowship holders benefiting since 2008.

Equality & Diversity (E&D) in the pursuit of research excellence are guiding principles for the MI. Both partners have active E&D committees, and E&D training, available to all staff, is mandatory for members of appointment panels. HWU, UoE and its School of Mathematics were all recipients of Athena SWAN Bronze awards for their commitment to supporting women's careers. We nonetheless recognise that women remain underrepresented among MI academic staff (10%) and are taking actions to remedy this: we are proactive in encouraging applications by women to new positions, are improving the representation of women in senior management, have adopted policies supporting those with family commitment, and are signatories to the LMS Good Practice Scheme for advancing women's career in mathematics. Smoktunowicz serves on the European Mathematical Society's Women in Mathematics Scientific Committee.

Research students

Our ambition is to attract the best students from across the world, to help them achieve excellence in research, teaching and knowledge exchange, and to provide the highest quality of training experience, ensuring success in their future careers. To achieve this, we have: invested substantial MI and university funds to increase the number of studentships we offer and overcome a contraction in EPSRC funding; intensified our recruitment efforts; and enhanced the training and support we provide. The result has been an expansion of the size of our graduate school (from 95 students in 2008-09 to 118 in 2012-13) and outstanding student successes (see below).

Funding. We offer a range of PhD scholarships: EPSRC DTA studentships (4-5/year, including 4 CASE since 2008), UoE Principal's Scholarships (2-3/year), SFC-funded ORSAS, HWU James Watt and Edinburgh Global scholarships (1-2/year). These have been supplemented by studentships funded by CANPDE, NAIS, SFRA and the MI. The flexibility of MI funding is used to offer partial scholarships and attract external funding. An increasing number of overseas students are supported by schemes from their country of origin (e.g. 4 China scholarships since 2008).

Recruitment. The MI receives a large number of online applications as a result of our research reputation and recruitment efforts including our participation in events such as the LMS-funded UK Prospects in Mathematics (held in Edinburgh in 2010). We specifically target top African students using our links to the African Institute for Mathematical Sciences (Schroers is an associate faculty member). Student excellence and research potential are the primary criteria for the allocation of funding; secondary factors include the mapping of research projects to the MI's research strategy and the support of early-career staff. A result of our recruitment policy is a highly international PhD cohort with 30-25-45% students from the UK, EU and rest of the world.

Training and support. All PhD students have two designated supervisors (for whom training is mandatory) who interact closely with them. Graduate School tutors offer pastoral support. Progress is monitored through regular meetings with the supervisors and Graduate School staff. Our PhD annual review system, in which students report in writing on their activities, provides more formal opportunities to discuss academic progress, training needs and intentions for the following year. A broad training in mathematics is provided through the Scottish Mathematical Sciences Training Centre (SMSTC), a consortium of Scottish universities supported by EPSRC in which the MI plays a central role (Carbery and Duncan have held the post of SMSTC Director since its inception), and by additional graduate courses. Throughout their programme, students attend about 6 weeks of transferable skills training, the core of which is the Generic Skills Course for Mathematics Postgraduates managed by ICMS for SMSTC. The rest is implemented through options that include UK GRAD schools, annual two-day training meetings run by the Edinburgh Mathematical Society, and a broad range of transferable-skill courses run by the universities. Our efforts in training are recognised: according to the most recent PRES survey, the MI has an 89% satisfaction rate for transferable-skill development, compared with a UK average of 72%. Each student is given a yearly travel allowance complemented by funding sources such as the Laura Wisewell fellowship or Roberts funding. We encourage our PhD students to gain experience in industry and we support internships offered through the Industrial Maths Knowledge Transfer Network. Almost all our students participate in teaching and outreach activities. They are leading MI activities such as weekly PhD student colloquia, the SIAM Student Chapter (founded in 2009), the University Maths Societies, and the Young Researchers in Mathematics meeting held in Edinburgh in 2013.

Student achievements. Our PhD students have won prizes and scholarships: the 2010 SIAM student paper competition (Bah), SIAM UKIE best student talk (Thompson 2011, Tzitzili 2013), the 2012 INFORMS Computing Society student paper award (Takac), 2013 IMA Leslie Fox Prize in Numerical Analysis (Takac, 2nd prize), 2009 LMS Cecil King Scholarship (Bellamy), LMS Best Student Presentation (Iliopoulou, 2013). They have been awarded fellowships including EPSRC Postdoctoral Fellowship and Oberwolfach Leibniz Fellowship (Bellamy). Many continue a research career, taking up post-doctoral positions in institutions such as Duke (Thompson), NYU (Karzhemanov), Johns Hopkins (Martinez Garcia), EPFL (Bah), McGill (Karzhemanov) or Cambridge (Bejan). Some who graduated since 2008 have already been successful in finding academic positions (Bellamy in Glasgow, Ziembowski in Warsaw, Ozkok in Southampton).

d. Income, infrastructure and facilities

The expansion of the MI's research activities since RAE2008 has been made possible by a marked increase in grant income: grant expenditure has increased from an average of £1.4M/year in the RAE2008 period to more than £2.7M/year in this REF period. The total award to the MI has more than trebled to exceed £22M since 2008. Our successful bids for the major infrastructure grants detailed below have been crucial for the enhancement of our research environment.

Grant-funded infrastructure
CANPDE. The Centre for Nonlinear Analysis and PDEs was established in 2007 to build research capacity in the rigorous study of nonlinear PDEs. It is funded by a £3.4M EPSRC S&I grant and a £300K SFC grant awarded to the MI. Together with its Oxford counterpart, it has invigorated PDE research in the UK through extended programmes of instructional workshops, crash courses and seminars, two major international conferences, and a visitor programme that brings some of the world's best analysts to Edinburgh (see www.maxwell.ac.uk/PDEs). The Centre has strengthened the MI's environment by funding 3 lectureships (Blue, Karakhanyan, Coutand), 3 RA positions and 3.5 PhD studentships since 2008, and by generating a vibrant programme of activities.

NAIS. The Centre for Numerical Algorithms and Intelligent Software was founded in 2009 and is supported by a £4.5M EPSRC Science and Innovation grant and £300K provided by SFC. It focuses on the development of new algorithms and compiler methodologies that exploit emerging computer architectures. The centre is a multidisciplinary collaboration between mathematicians in the MI and at Strathclyde University with computer scientists at UoE, the Edinburgh Parallel Computing Centre (EPCC) and a variety of end users. Permanent lectureships have been established in 4 of the partner departments (Loisel, Maddison and Richatrik in the MI), and a wide range of PhD and postdoctoral research projects are underway. NAIS supports visitor and workshop activity across a network of UK partner universities (see www.nais.org.uk). Thanks to the MI's leading role, there is a concentration of activities in Edinburgh which benefits the Applied & Computational Mathematics, Operational Research, and Analysis & PDE groups.

SFRA. The Scottish Financial Risk Academy is a consortium led by the MI and supported by major financial institutions (Aberdeen Asset Management, Barrie & Hibbert and Lloyds Banking Group) that addresses risk management in the financial industry (see www.sfra.ac.uk). Launched in 2010 with £0.3M seed funding awarded by SFC, it is now supported by its industrial partners who committed £200k over the next 3 years. The academy runs Knowledge Exchange activities that stimulate interactions between the academic sector and the financial services industry: biannual Risk Colloquia, Knowledge Transfer Workshops, placements for MSc and PhD students, and postgraduate courses taught by industry professionals. Our capacity in the financial area has been further enhanced by the establishment of the Actuarial Research Centre (ARC) with funding from the Faculty of Actuaries Endowment Fund supporting 2 full PhD positions/year from 2013.

ICMS is a vital part of the research environment in Edinburgh and for the whole UK mathematical sciences community (see www.icms.org.uk). As one of the major centres hosting international mathematics meetings (such as INI, Oberwolfach, MSRI) it is widely acknowledged for the impact of the high-level workshops it organises. Since 2008 over 70 workshops have been held, attracting speakers of the stature of Atiyah, Donaldson, Smale, Tao, Witten (all Fields Medallists), and Higgs (Nobel laureate). Its success is recognised by EPSRC which has awarded the MI £3M for ICMS since 2008. This rewards the continuous investment made by the MI, both financially (£50K/year) and through the many contributions of its staff to management, fundraising and workshop activities. In 2010, ICMS moved to a well-appointed building in the centre of Edinburgh incorporating 100-seat auditorium. This enables ICMS to host larger events and facilitates its Research-in-Groups programme. Since 2008 ICMS has strengthened its interactions with industry by employing a dedicated Knowledge Transfer Officer. It has expanded its PE programme, organising events in the annual Edinburgh Science Festival and public lectures by pre-eminent figures to make a major contribution to the public visibility of mathematics in the UK. ICMS brings many benefits to the MI: its facilities are regularly used for MI group meetings, seminars and public lectures; all MI researchers can attend its workshops and other meetings, enabling them to interact with the continuous flow of leading mathematicians these attract. ICMS supports the wider mathematical community by administering the SMSTC PG courses, organising externally funded conferences, and managing mathematics journals.

Other infrastructure grants held in the census period are: an £1.8M STFC grant awarded jointly to the MI and to theoretical physicists at UoE which supports their activities in mathematical physics under the auspices of the Tait Institute; and an EPSRC grant funding the ERPem programme Bridging the Gaps Between Engineering and Maths in ERPem (2007-2010).

Grant portfolio and strategy

In addition to the grants funding the infrastructure described above, the MI holds a large portfolio of grants supporting specific research projects (93 in the REF period). Our improved grant-application

process, which includes systematic internal reviews, has proved effective, with 64 grants worth more than £100K active in the REF period (vs 36 in the previous 6-year period). We have achieved notable successes with fellowships (see section c) and taken full advantage of the EPSRC first-grant stream (7 awarded since 2008) and responsive-mode grants (39 awarded since 2008).

A key aspect of our strategy is the diversification of our grant portfolio: responding in part to the reduction in EPSRC funding for the mathematical sciences, staff have been encouraged to seek funding beyond EPSRC's mathematics programme. The result has been the considerable broadening of the spectrum of our funders: we have secured substantial grants from the EC (Bayer, Hansen, Leimkuhler, Zachary, Foss, Smoktunowicz), BBSRC (Painter, Sherratt), NERC (Sherratt, Vanneste), Royal Academy of Engineering (Mackay), Leverhulme Trust (Painter, Konechny, Sherratt, Szabo), National Philanthropic Trust (Antal), EPSRC Digital Economy (Gondzio, Richtárik, Buke), EPSRC Living with Environmental Change (Gibson), National Institute for Standards & Technology (Olde Daalhuis), NSF (Bayer, Hering, Leimkuhler, Sierra), MOD (Richtárik). Increasingly, industrial and financial partners provide sources of research income for the MI: Accelrys (Leimkuhler), HEG and Intel (Gondzio) and Format International and SAS (Hall), as do consultancy contracts (Xi Engineering (Bochkina), Dakota Consulting (Olde Daalhuis), Longevitas/SJ Richards (Currie)). Our awards from these sources since 2008 exceed £4M.

Furthering the diversification strategy is a priority of the MI for the coming years: it will be a key to the resilience of research activities in what is expected to remain a challenging financial climate. The steps taken to promote multidisciplinary research and collaborations with non-academic partners, for instance NAIS and ICMS's appointment of a KT officer (see REF3a), will contribute to the sustainability of the MI's activities.

Facilities

The MI staff and PG students are accommodated in two well-equipped buildings. Cross-campus interactions are supported by hot-desking facilities in each, by a coordinated timetabling of events, and by our policy of reimbursement of taxi costs for all travel between the sites. We enjoy exclusive use of 3 video-conference rooms reserved for MI-specific activities (e.g. broadcasting of seminars), PG training and interactions with external collaborators. Among our excellent general facilities are the large libraries subscribing to a broad range of journals, and a new centre dedicated to postgraduate learning that provides lecture theatres, study rooms and social space. Our expanding needs for high-performance computing are well catered for: our large university clusters, including the ECDF 3000-core cluster and GPU cluster, are used on a regular basis by the Applied & Computational and Operational Research groups. The MI runs its own computer network with several high-end machines and a small GPU. In addition to these physical facilities, we benefit from Edinburgh's position as a world-leading centre in high-performance computing, with the Edinburgh Parallel Computing Centre (EPCC) hosting the UK's main scientific supercomputer HECToR as well as several other high-end facilities. EPCC is directly associated with the MI through its participation in NAIS and further benefits MI staff through its training and visitor programmes.

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e. Collaboration or contribution to the discipline or research base

Collaborations. Virtually all MI staff are engaged in international collaborations. These have resulted in hundreds of research visits over the census period, both to and from the MI. Several staff also take leading roles in large-scale collaborative projects: Gordon is a CoI for the Network Representation Theory Across the Channel (funded by two consecutive EPSRC grants 2008-12, 2012-15 and twinned with CNRS and DFG networks in France and Germany); Foss and Zachary were team leaders in the European Network of Excellence EURONFI-EUROFGI-EURONF (2003-2012); Vanneste was PI of the EPSRC Network Wave-Flow Interactions (2008-11); Leimkuhler co-

authored the bid for the EPSRC Network Mathematical Challenges of Molecular Dynamics (2008-11); Boulton co-authored the bid for the EPSRC Network Matrix and Operator Pencil Network (2009-12); Lord is PI of the EPSRC Network Porous Media Processes and Mathematics (2012-15).

Interdisciplinary research. We recognise that collaboration beyond the mathematical sciences is crucial to maximising and sustaining the impact of our research. We see the increasing embedding of MI staff in application disciplines as a very positive development which enhances our capacity to generate research income from sources beyond those most frequently accessed by the mathematical sciences. The main sectors with which we interact are now described.

Collaborations with researchers in **Biology and Medicine** include Antal's work on metastasis in pancreatic cancer (published in *Nature*) and on the evolutionary dynamics of cancer, Dindos's work on biofilms, and Sherratt and Painter's work on cell differentiation and pattern formation in embryos. Gibson has worked with clinicians and microbiologists on control strategies for hospital infections and with plant scientists on the control of arboreal pathogens. In **Ecology**, White's collaborations include modelling phenomena such as tolerance to parasites (in *PNAS)* and the evolution of diversity, while Leinster's develop measures of ecological diversity. MI staff collaborate extensively with the **Engineering and Physical Sciences**. High-profile work includes Leimkuhler's collaborations on molecular modelling supported by the NSF/EPSRC programme Software for Sustained Innovation in Chemistry. Gibson has collaborated with building engineers to model the energetics of buildings under climate change. Our Probability & Statistics group works with power engineers on buffering, storage and stability in energy supply systems (Foss, Zachary, Cruise). Our research on the simulation of porous media flows (Lord, Duncan, Banjai) involves extensive collaboration with geoscientists and petroleum engineers. Our Actuarial & Financial Mathematics group has long enjoyed close collaborations with the **Financial and Insurance industries** providing solutions to problems in asset and risk management, and mortality projections (see REF3a). Highlights include McNeil's work on undercapitalisation of the banks and Donnelly's analysis of the causes of the sub-prime mortgage crisis. The MI's close relationship with key stakeholders has been substantially enhanced through the establishment of the industry-funded SFRA and ARC. In recognition of their impact in business and finance, 8 MI staff are being submitted to UoA 19 in the REF. Similarly, Aitken's submission to UoA 20 recognises the influence on Law of his pioneering work applying statistics to forensic sciences. We have taken several initiatives to stimulate further collaborations: our Maximaths programme has so far funded 15 small projects pairing up mathematicians with colleagues in other disciplines; we lead EM^3, a grouping stimulating interactions between departments through its workshop and visitor programmes.

Exemplars of leadership. The international reputation of our academics gives them many opportunities to lead or influence the research community. The examples below give an indication of the breadth and depth of their contributions.

MI staff have been rewarded by **prizes** and **awards**. The most significant prizes received are: European Mathematical Society Prize (Smoktunowicz, 2008), Adams Prize (Vanneste, 2010), Whittaker Prize (Smoktunowicz, 2009), and Polish Prime Minister Prize for Habilitation (Smoktunowicz, 2008). The most notable fellowships received include Fellowships of the American Mathematical Society (Smoktunowicz, 2012, Atiyah 2012), Fellowships of the Royal Society of Edinburgh (Smoktunowicz, 2009, Gordon 2010, Wright, 2011).

Our staff have been invited for numerous **research visits**. This includes several invited professorships: Emmy Noether Professor at University of Göttingen (Smoktunowicz, 2011), ETH Zurich (Gordon, 2012), MPI Bonn (Ranicki, 2010), Ecole Normale Supérieure (Foss, 2010), EURANDOM Chair of the Year (Foss, 2011-12), Queensland University of Technology (Gibson, 2009-12), IAS Princeton (Simón, 2012), IPMU Tokyo (Figueroa-O'Farrill 2009), KITP (Simón

2012). Staff have made extended research stays in all the top international centres for mathematical research (INI, MSRI, Fields Institute, Oberwolfach, IAS, IHES, KITP, Institut Mittag-Leffler, Simons Center for Geometry and Physics, IPMU, Perimeter Institute, Erwin Schrödinger International Institute, IPAM Los Angeles).

The many **plenary and keynote lectures** given by MI staff gives further evidence of our international recognition. These include invited lectures at the International Congress of Mathematicians (Gordon, 2010), at European Congress of Mathematics (Smoktunowitz 2008). Others are: Europt (Gondzio, 2013), Stochastic Programming XIII (Gondzio 2013), New Frontiers in Applied Probability (Foss, 2011), ICM-2010 Satellite conference on Harmonic Analysis (Carbery 2010), Mathematics of String Theory Special Session at PRIMA 2013 (Szabo 2013), Cycles, Calibrations and nonlinear PDEs, Simons Centre (Figueroa-O'Farrill), 6th ERCIM International Conference on Computational & Methodological Statistics (Bochkina 2013).

Our contributions to the research community include the organisation of numerous **conferences, meetings** and **workshops**. A major effort was the hosting of the 2010 joint BMC/BAMC meeting in Edinburgh. Other notable contributions by MI staff include: the INI programme Stochastic Processes in Communication Sciences (Foss, 2010), the conference Atiyah 80 (Ranicki, 2008), the MSRI programme "Representation Theory and Noncommutative Algebraic Geometry" (Gordon, Sierra, 2012-13), SIAM Conference on Nonlinear Waves and Coherent Structures (Beck, 2012), Operators and Operator Algebras in Edinburgh (Carbery, 2009), Corfu Summer Institute (Szabo, 2011 & 2013), EPSRC Symposium Capstone Conference (Leimkuhler, 2009), Nanostructured Thin Films IV, V, VI (Mackay, 2011-13), Complex Analysis and PDEs at BIRS (Wright, 2010), Approximation theory at CRM Barcelona (Wright, 2011), NSF-NAIS Workshop on Intelligent Software (Leimkuhler, 2009), Multiscale Molecular Modelling (Leimkuhler, 2010) and LMS Invited Lecture series (Wright, 2009, Chelstov, 2013). We regularly co-organise workshops in international research centres including at Oberwolfach (4 since 2008), INI (4), MSRI (3), Fields Institute, Luminy, Banff, KITP and ICMS (more than 40).

Our staff contribute to **learned societies**, **funding bodies** and **professional organisations**. Examples are: member of the EPSRC Strategic Advisory Team for Mathematics (Gordon), members of the REF2014 subpanel (Gordon, Gondzio), President of the EMS (Carbery), members of NSF panels (Beck, Bayer, Foss, Ranicki), mathematical editor of the Digital Library of Mathematical Functions (Olde Daalhuis), Project Manager for the COIN-OR Simplex Solver (Hall), Chair of working groups of the European Exascale Software Initiative (Grothey), member of AgreenSkills Scientific Committee (Gibson), UK member of the Management Committee of the COST Action on Mathematical Optimization in the Decision Support Systems for Energy Networks (McKinnon), member of the LMS Prizes Committee (Carbery), member of the SIAM Dahlquist Prize Committee 2009 (Leimkuhler), member of the Scientific Advisory Board of Imperial College's MRC Centre on Outbreak Analysis and Modelling (Gibson). We also make a permanent major contribution to the management of the ICMS.

Finally we act as **editors** or associate editors for more than 50 international journals including Bulletin of the AMS (Ranicki), Proceedings of the LMS (Gordon and Wright), Proceedings of the Royal Society of London B (Gibson), Classical and Quantum Gravity (Simón), Advances in Applied Probability (Foss), Markov Processes and Related Fields (Foss), Computational Optimization and Applications (Gondzio), Mathematical Programming Computation (Gondzio, Grothey), SIAM Journal on Numerical Analysis (Leimkuhler), Nonlinearity (Leimkuhler), SIAM Journal on Scientific Computing (Lord) and Journal of Physics A (Szabo). Several MI staff also contribute editorial and management roles for the Proceedings of the Edinburgh Mathematical Society and the Proceedings of the Royal Society of Edinburgh.