Institution: University of Glasgow

Unit of Assessment: 10 – Mathematical Sciences

Title of case study: Mathematical models for design and stress analysis in the rubber and automotive industries

1. Summary of the impact

Professor Ray Ogden FRS has made fundamental advances in mathematical models for the elastic response of rubber-like materials. These models have been adopted as the standard starting point for the design and analysis of rubber-like solids and they have been incorporated into the industry-standard commercial `finite element' software packages, including Abaqus, ADINA, ALGOR, ANSYS and MARC. These packages are widely used by professional engineers for design calculations and stress analysis, particularly in the aeronautical and automotive industries, playing a crucial role in design decisions associated with enormous financial investments and with the safe and successful operation of the products involved. Models have now also been constructed for the behaviour of soft biological tissue and these are in widespread use in applications in cardiovascular research and the life sciences.

2. Underpinning research

Rubber and other rubber-like materials are ubiquitous in the manufacture of machinery and equipment. Understanding the properties of these materials when they are placed under stress in routine, and indeed abnormal, use is therefore vital in the design and manufacture of machinery and equipment which is able to perform safely to a desired specification. Professor Ray Ogden (Professor, George Sinclair Chair 1984-2010, Visiting Professor 2010-2012, George Sinclair Chair of Mathematics 2012-present) has made fundamental advances in modelling the elastic response of rubber-like materials through the development of strain-energy functions, which have been used both in further theoretical developments and in a very wide range of applications.

A recent key advance, building on earlier work, has provided models for *inelastic stress-softening* effects in rubber-like materials using a novel theory of *pseudo-elasticity* [1]. When a rubber test piece is loaded in simple tension from its virgin state, unloaded and then reloaded, the stress required on reloading is less than that on the initial loading. This *stress-softening* phenomenon is referred to as the Mullins effect. Prof. Ogden developed a model for this, based on the theory of incompressible isotropic elasticity but amended by the incorporation of a single continuous deformation dependent parameter, interpreted as a damage parameter. This parameter controls the material properties in the sense that it governs the material response by a strain-energy function, which switches continuously from one form to another when loading is switched to unloading and *vice versa*. For this reason the model is referred to as pseudo-elastic. Energy dissipation in a loading-unloading cycle is measured by a damage function which depends on the damage parameter. A specific form of this function with two adjustable material constants, coupled with standard forms of the strain-energy function, is used to model the qualitative features of the Mullins effect in both simple tension and pure shear. This work has become a standard reference for subsequent models and it has received many citations in the scientific literature (Google Scholar: 283, World of Science: 185).

More recently still, and in collaboration with Professor Gerhard Holzapfel (Graz University of Technology, Austria), Prof. Ogden has developed models for the nonlinear elastic behaviour of soft biological tissues [2, 3], in particular for artery wall tissue. For example, some arteries can be modelled as thick-walled nonlinearly elastic circular cylindrical tubes consisting of two layers (corresponding to the *media* and *adventitia*). Each layer is treated as a fibre-reinforced material with the fibres corresponding to the collagenous component of the material. A specific form of the model, which requires only three material parameters for each layer, is used to study the response of an artery under combined axial extension, inflation and torsion. The characteristic and very important residual stress in an artery *in vivo* is accounted for by assuming that the natural (unstressed and unstrained) configuration of the material corresponds to an open sector of a tube, which is then closed by an initial bending to form a load-free, but stressed, circular cylindrical configuration prior to application of the extension, inflation and torsion. The effect of residual stress on the stress distribution through the deformed arterial wall can then be described. These models have been used extensively for the analysis of the mechanical behaviour of a variety of soft tissues and they have provided an important springboard for further modelling developments. The scientific importance of this work is reflected in the very high level of citations for [2] (Google Scholar: 1206, Web of Science: 752) and [3] (Google Scholar: 502, Web of Science: 284) in the literature.

3. References to the research

[1] R.W. Ogden and D.G. Roxburgh, A pseudo-elastic model for the Mullins effect in filled rubber. Proceedings of the Royal Society of London A 455 (1999), 2861-2877. doi.org/10.1098/rspa.1999.0431 *

[2] G.A. Holzapfel, T.C. Gasser and R.W. Ogden, A new constitutive framework for arterial wall mechanics and a comparative study of material models. Journal of Elasticity 61 (2000), 1-48. [Reprinted in Cardiovascular Soft Tissue Mechanics, edited by S. C. Cowin and J. D. Humphrey, Kluwer Academic Publishers, Dordrecht (2001).] doi:10.1023/A:1010835316564 *

[3] T.C. Gasser, R.W. Ogden and G.A. Holzapfel, Hyperelastic modelling of arterial layers with distributed collagen fibre orientations. Journal of Royal Society Interface 3 (2006), 15-35. doi:10.1098/rsif.2005.0073 *

* best indicators of quality

4. Details of the impact

Engineering applications that require the investigation of material properties use software packages which make use of `finite element' (FE) methods. These packages are used extensively worldwide and considered indispensible by professional engineers who undertake design calculations and stress analysis, particularly in the automotive and aerospace industries where rubberlike materials are employed in many components. Such materials can deform significantly, and the underlying mathematical models therefore play a crucial role in design decisions which are central to the safe and successful operation of the end products, and associated with substantial financial investments.

There are several industry-standard commercially available systems, including Abaqus, ADINA, ALGOR, ANSYS and MARC. All of these systems incorporate Prof. Ogden's models as core standard tools for the analysis of hyperelastic behaviour and the properties of rubberlike materials.

Given the widespread use of materials of this type in manufacturing and the very high volume of associated engineering design and analysis activity, the industrial impact of these models is difficult to quantify precisely but is clearly enormous. The methods are key in automotive and aeronautical engineering and, from these industries alone, the economic impact is therefore extremely high.

The software designers of Abaqus, ADINA and ANSYS, the three most widely used FE software packages, adopted the Ogden-Roxburgh model from published literature and incorporated it into their systems. (In particular, the Abaqus system incorporates all the Ogden models.) These FE packages are used for calculations involving solid materials in mechanics problems, and the models are essential to these calculations because they give the fundamental constitutive relationships needed to represent the properties of the materials proposed for use in the product being designed. These software packages, all utilising the Ogden models, are standard core tools in industrial design.

Abaqus Unified FEA is a suite of simulation software in the SIMULIA brand of Dassault Systèmes, an international company with over 11,000 employees with headquarters in Paris, France. Dassault Systèmes is one of the industry leaders in the provision of tools to facilitate realistic simulation in product design and manufacturing, which reduces manufacturing costs by cutting design time and prototype costs. The Abaqus manual gives comprehensive and detailed descriptions of the methods available for implementation. Chapter 18 deals with `Elastic mechanical properties' and section 18.5 with `hyperelasticity'. In section 18.6.1 where the `Mullins effect in rubberlike materials' is discussed, the methods described are based on the Ogden-Roxburgh model and [1] is the only referenced paper. Under `Anisotropic hyperelastic behavior' the Holzapfel-Gasser-Ogden model is one of only two specific options listed. Similar evidence of the impact of these models is provided in the manuals of the other major finite element systems. It is therefore clear that professional engineers worldwide are making extremely wide use of these methods to solve a whole variety of problems in a very wide range of application areas.

Two quotes from Endurica [1], an industrial consulting and software company:

> *"Although I had been aware of several alternative approaches to modeling the [Mullins] effect prior to encountering your model, I found all of these to be out of reach for our applications for various reasons. I was immediately impressed by the simplicity of the Ogden-Roxburgh model, its compatibility with approaches we were already investing in, and its effectiveness in accurately representing the main features of the effect. Recognizing these advantages, and what they would mean to modeling efforts in the tire industry and beyond, I was able to persuade my management to fund a project with HKS (now Dassault Systemes) to implement the Ogden-Roxburgh model in the Abaqus Finite Element code."*

> *"The Ogden-Roxburgh model is continuing to grow in its impact. I have noticed other workers building various additional effects on top of the Ogden-Roxburgh model (permanent set and cyclic softening, for example). For my part, I have implemented the Ogden-Roxburgh model to represent rubber's cyclic stress-strain behavior in the world's first commercially available fatigue analysis software."*

From Abaqus [2]:

> *"This [the Ogden-Roxburgh] model was also implemented in Abaqus and serves as the only way to capture this Mullins effect, stress softening behavior, especially noticeable in filled rubber. Capturing this stress softening behavior in elastomers eliminates a key approximation that FEA users have had to make for many years, and leads to a much higher fidelity material model, and thus a much more realistic simulation model."*

From ANSYS [3]:

> *"Without this material model, accurate representation of the rubber behavior is very difficult. Because of this, it is a popular practice for rubber product design in the ANSYS user community."*

From Abaqus [2] again:

> *"More recently, another significant contribution from Professor Ogden and colleagues made its way into the commercial Abaqus FEA software. This was a proposed model for capturing the anisotropy in biological soft tissues. This micromechanically based anisotropic strain energy potential (Holzapfel, Gasser, and Ogden, 2000 and also, Gasser, Ogden, and Holzapfel, 2006) is now available in Abaqus for modeling biological tissues and other anisotropic elastomeric materials. This anisotropic strain energy potential is commonly used by life sciences researchers."*

5. Sources to corroborate the impact

Statements from the following companies [1, 2, 3] are available from HEI:

[1] Endurica LLC was founded in 2008 to provide services, technology, and training that accelerate reliable design for elastomer materials and components:
www.endurica.com
(Statement from President of company)

[2] Abaqus Analysis User's Manual (version 6.8). Vol. III: Materials. Dassault Systèmes Simulia Corp., Providence, RI, USA, 2008:
http://www.3ds.com/products-services/simulia/portfolio/abaqus/overview/
(Statement from Senior Sales Manager, EMEA Academia)

[3] ANSYS, ANSYS, Inc., Canonsburg, PA, USA:
http://www.ansys.com/Products
(Statement from Lead Product Manager, Mechanical Products)