Institution:	**University of Manchester (UoM)**
Unit of Assessment (UoA):	**B10 (Mathematical Sciences)**

Title of case study:
Mathematical Software for the Nearest Correlation Matrix

1. Summary of the impact

Correlation matrices play a key role in financial modelling, but their empirical construction (based on the actual statistical data) may lead to negative variances, which can lead to complete failure of a model. Our research has resulted in algorithms for efficiently computing the unique *nearest correlation matrix* (NCM) that does not yield negative variances. The most direct impact is to Numerical Algorithms Group (NAG) Ltd, whose library sales and renewals have been increased by an estimated £250k following the inclusion of our NCM codes. Further impact is to NAG clients, including the Tier 1 Investment Banks, with at least six of the top ten [e.g., Credit Suisse and Morgan Stanley] known to be using the new NAG nearest correlation matrix codes, leading to improved reliability of their financial models.

2. Underpinning research

The impact is based on research that took place in the unit of assessment from 2000-date, with the first major publication in 2002. The key researchers were
Professor Nick Higham (2000-date).
Dr Rudiger Borsdorf (MSc student and PhD student, 2006-2012).

The correlation matrix is a classical concept from statistics and specifies the degree of linear dependence between various random quantities (e.g. various financial assets). The aim of the research was to develop efficient algorithms for computing the nearest correlation matrix to an arbitrary matrix. The algorithms on which this case study is based are:

1. The alternating projections algorithm [1]. This is the first algorithm proven to compute the global minimizer of the distance to the set of correlation matrices.
2. The preconditioned Newton algorithm [2], which takes the Newton method derived in [Hou-Duo Qi and Defeng Sun, A quadratically convergent Newton method for computing the nearest correlation matrix, SIAM J. Matrix Anal. Appl., 28(2):360-385, 2006] and constructs an efficient and reliable algorithm, by using preconditioned iterative solution of the Newton equations, carefully avoiding roundoff problems in the line search, and making other improvements explained in [2].

3. References to the research

The research has been published in leading high impact numerical analysis journals.
Citations are shown for the Web of Science (WOS) and Google Scholar (GS) as of 9-8-13. Reference [1] has been the most downloaded full-text PDF in IMA J. Numer. Anal. every year since 2008 (source: IMA Journal of Numerical Analysis Publisher's Reports, OUP, 2009-2012).

[1] N. J. Higham, Computing the nearest correlation matrix - A problem from finance. IMA J. Numer. Anal., 22(3):329-343, 2002. DOI 10.1093/imanum/22.3.329. [WoS: 121, GS: 304].
[2] R. Borsdorf and N. J. Higham, A preconditioned Newton algorithm for the nearest correlation

matrix. IMA J. Numer. Anal., 30(1):0 94-107, 2010. DOI 10.1093/imanum/drn085. [WoS: 9, GS: 25].

4. Details of the impact

Context

Correlation plays a fundamental part in any financial model dealing with more than one asset e.g. CAPM, Markowitz portfolio theory, the LIBOR market model, or any multi-asset extension of the various market models used. However estimating correlation is notoriously difficult: in practice market data is often missing or stale; different assets are sampled at different time points (e.g. some daily and others weekly); and the data may even contain arbitrages due to averaging of bid and offer quotes. As a result, estimated correlation matrices are frequently not positive semidefinite, with the consequence that variances are negative, which is forbidden by definition. There is a real need to correct the non-positive semi-definiteness of estimated correlation matrices, while at the same time staying true to the correlations implied by the market data - in other words, not changing the estimated matrix too much, but just enough to make it mathematically sound. This is what the NCM routines do, and this is why they are crucial in the financial industry and in many other contexts.

Prior to our research, ad-hoc methods had been developed in an attempt to compute the nearest correlation matrix, but none were guaranteed to compute it and some were not even guaranteed to converge. Our work has provided fast and reliable algorithms for computing the unique nearest correlation matrix in the Frobenius norm.

Pathways to Impact

The Numerical Algorithms Group (NAG) Ltd. has been a world leader in the development and distribution of numerical software for more than 40 years, and has offices in Oxford and Manchester and subsidiaries in Chicago, Tokyo and Taipei. Higham has long-standing professional relationships with colleagues from NAG going back to the 1980s. As a result of these links, the research reported here has been strongly influenced by the needs of NAG and indeed NAG provided partial funding for Borsdorf's PhD studies in Manchester. The Manchester researchers have assisted in translation of the algorithm in [2] into NAG software, and NAG has been in a position to rapidly incorporate the software into their products.

In order to maximise impact in the financial sector, NAG is devoting significant effort to promote the developed software using marketing material, brochures [S1], web videos, targeted site visits and publication via seminars and Trade shows. Higham has joined NAG representatives on visits to BNP-Paribas (London) and Barclays Capital (London), both in June 2011, at Standard and Poors (New York, December 2011), and at Credit-Suisse and Morgan Stanley in March 2013. He gave seminars on the NCM problem at all these venues and at the Institute of Actuaries in March 2013.

In addition, the algorithm of [1] is freely available in implementations in MATLAB, R, and SAS.

Reach and Significance of the Impact

The key user base is the financial industry and at least six of the top ten Tier 1 Investment Banks are using NAG nearest correlation matrix (NCM) codes [S3] that implement the preconditioned Newton algorithm [2]. The codes can be directly incorporated into customers' existing financial models because they can be called from Fortran or C, from MATLAB via the NAG Toolbox for MATLAB, and from Excel. In particular, the link to Excel allows the research to reach practitioners

who are not necessarily programmers, such as those from the actuarial community [S2].

Due to the importance of the codes, and based on feedback from NAG customers, NAG has worked with Higham and Borsdorf to improve the codes, gaining a factor two increase in the speed since their first introduction [S4].

The nearest correlation matrix codes in the NAG Library are helping NAG to gain more revenue across all sectors, both from new customers and from existing users who are more likely to renew with the new features. The estimated total additional income to NAG as a consequence of the inclusion of our codes into the library (new licenses plus renewals) in the period January 2010 - July 2013 is £250,000 [S3]; this is very significant for a company with about 70 FTE staff.

Commercial sensitivity means that the Tier 1 Banks are unwilling to disclose any revenue increases as a consequence of using these algorithms, but, for context, Morgan Stanley report assets of US$347 billion under management or supervision (June 30, 2013) [S5], and Credit Suisse report CHF 408 billion under management at the end of 2011 [S6]. Thus, if used in management of 1% of the assets (a conservative estimate) our algorithms would still inform financial decisions on the scale of billions of dollars.

5. Sources to corroborate the impact (indicative maximum of 10 references)

[S1] Nearest Correlation Matrix brochure
www.nag.co.uk/IndustryArticles/Nearest_Correlation_Matrix.pdf, 23 July, 2010
(Evidence of promotion by NAG of the software based on our algorithms)
[S2] John Holden and Jacques duToit, Numerical software & tools for the actuarial community, www.nag.co.uk/market/seminars/nag_actuarial_community_sep2012.pdf; accessed 12-1-13.
(Demonstrates that research is being made available to actuarial community)
[S3] Letter from Vice President for Sales, NAG, July 22, 2013.
(Supports finanical claims about NAG)
[S4] Matrix functions, correlation matrices, and news from NAG, www.nag.co.uk/Market/events/craig_nag_wilmott_2011.pdf, talk at NAG Quant Event, London, by Dr Craig Lucas, 2011; accessed 9-8-13. (Demonstrates increased speed of the code)
[S5] http://www.morganstanley.com/about/ir/earnings_releases.html
(Reports assets managed by Morgan Stanley)
[S6] https://www.credit-suisse.com/who_we_are/en/asset_management.jsp
(Reports assets managed by Credit Suisse)