Institution: University of Birmingham

Unit of Assessment: Mathematical Sciences

Title of case study: Efficient Development and Assessment of Extraction Strategies in the Petroleum Industry

1. Summary of the impact (indicative maximum 100 words)

This case study reports the development of a new approach to solving full field reservoir problems with inhomogeneous and anisotropic permeability and variable reservoir This comprehensive body of work arose from discussions between scientists at the Schlumberger Technology Centre, Abingdon, and the internationally recognised Nonlinear Waves group in the School of Mathematics, and has been supported under two contracts with Schlumberger Oilfield UK PLC through their Technology Centre in Abingdon UK. The work has provided Schlumberger with a fast, robust and efficient tool for the rapid assessment of optimisation problems relating to oil well location sites in new oil reservoirs, and has been implemented in their recently developed GREAT facility for reservoir estimation and analysis. Schlumberger PLC is an international company which plays a premier role in supplying the petrochemical industry with services such as seismic acquisition and processing, well testing and directional drilling, flow assurance and extraction strategy. The work described in this case study took place from 2007 to 2011, and involved D J Needham (University of Birmingham) and S Langdon (University of Reading).

2. Underpinning research (indicative maximum 500 words)

Here, we describe the two-dimensional problem. The full three-dimensional problem has been dealt with in subsequent work. We introduce the parameter E = *h/l* and consider asymptotic solutions to the equations of motion of the fluid in increasing powers of E, with $0<E<< 1$. In the vicinity of a well (the *inner* region) the pressure field is two-dimensional, but away from the wells (the *outer* region) the pressure field is only one-dimensional. This immediately leads to a reduction in complexity. Here, however, rather than solving the full equations of motion numerically in the inner and outer regions, we construct two-term expansions in both the inner and outer regions. The expansions in the inner and outer regions can then be matched, via the Van Dyke asymptotic matching principle, enabling us to derive amenable analytical expressions for all significant process quantities.

We begin by deriving the equations of motion in the porous medium. Conservation of mass and momentum lead to a strongly parabolic linear initial boundary value problem for the dynamic fluid pressure (from which the fluid velocity field can be deduced), with Neumann boundary conditions, under the assumption that the walls are impenetrable to the fluid in the porous medium. This initial boundary value problem has a unique solution, but its direct computation would be expensive, primarily due to stiffness when $0 < E << 1$. We thus consider the associated steady state problem [SSP], a linear strongly elliptic Neumann problem, which also has a unique solution (up to a constant) under the further constraint that the sum of the total volume fluxes at the wells (the line sources and sinks) is zero. Solution of the steady state problem is then considered. Subtracting the solution of the steady state problem from the solution of the initial value problem leads to a strongly parabolic homogenous problem with no discontinuities across the sources and sinks. The solution of this problem leads to a regular self-adjoint eigenvalue problem [EVP] whose solution is considered.

Rather than solving [SSP] and [EVP] directly, the solution to each problem is considered in the asymptotic limit $E \to 0$, via the method of matched asymptotic expansions. For the two-dimensional problem these asymptotic solutions can be constructed analytically. To solve [SSP], we proceed first with the situation when the wells are well spaced and are away from the reservoir boundaries, after which the case of wells close to a boundary, or close together, is considered. The asymptotic solution can be constructed directly in the outer region, up to $O(E^2)$. In the inner region, determination of the leading order terms reduces to the solution of a strongly elliptic problem whose solution can be written analytically in terms of the eigenvalues and corresponding eigenfunctions of a regular Sturm-Liouville eigenvalue problem. The asymptotic solution of [EVP] also reduces to a regular Sturm-Liouville eigenvalue problem identical in structure to that discussed earlier and a consideration of this allows us to demonstrate that the solution to the full initial boundary value problem approaches the solution to the steady state problem through terms exponentially small with respect to time t as $t \to \infty$. With Dz being the permeability scale in

the vertical direction and Dx being the permeability scale in the horizontal direction, the further generalisation that $Dz = o(Dx)$ rather than O (Dx) is considered, where it is shown that the structure of the solution is identical to that found for the case that $Dz = O(Dx)$, after a suitable redefinition of the parameter E. The constraint on the sum of the total volume fluxes at the wells being zero is removed at a later stage.
The development of the two-dimensional theory into a three-dimensional theory is straightforward and has been completed in detail. Finally a fast and very efficient numerical implementation has been developed and delivered as a tool to Schlumberger. Aspects of this tool have been incorporated into the GREAT facility recently developed by Schlumberger.

3. References to the research (indicative maximum of six references)

‘The unsteady flow of a weakly compressible fluid in a thin porous layer I: Two-Dimensional Theory’, SIAM Jl Appl Math, 2009, 64, 4, 1084-1109. (D J Needham, S Langdon, G S Busswell*, J P Gilchrist*)

‘The unsteady flow of a weakly compressible fluid in a thin porous layer II: Three-Dimensional Theory’. Q Jl Mech Appl Math, 2013, 66, 1, 97-122. (D J Needham, S Langdon, B A Samson*, J P Gilchrist*)

‘The unsteady flow of a weakly compressible fluid in a thin porous layer III: Three-Dimensional Computations’. Q Jl Mech Appl Math, 2013, 66, 1, 123-155. (S Langdon, D J Needham, B A Samson*, J P Gilchrist*)

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4. Details of the impact (indicative maximum 750 words)

It is standard practice in the oil and gas industry to use reservoir simulators based on numerical methods such as the finite difference or finite element techniques. This kind of approach has been shown to be enormously successful over the years in modelling a wide variety of physical processes in the reservoir e.g., faults, rock layering effects, complex fluid phase behaviour, etc.

While reservoir simulators of this type will continue to play a crucial role in the industry, it is well known that to use them takes considerable expertise and time. Because of the numerical nature of the modelling process, gridding, time stepping, stiffness and convergence issues require care and attention. Extremely long execution times are often necessary for certain types of problems, e.g., hydraulically fractured wells, and to maintain numerical stability.

Analytical techniques, for the reason outlined above, can therefore play a valuable role in the industry. Such techniques, although they may have some simplifying assumptions, allow a reservoir or production engineer to perform a quick and reliable study of their reservoir in order to obtain a broad understanding of the dynamical processes and make approximate costing forecasts. Moreover, analytical solutions are extremely fast to compute and provide none of the stability, time stepping and convergence issues seen with a numerically based simulator. Also, a necessary step in many reservoir studies involves the history matching of observed data by optimizing model parameters. The history matched model is then used for performance prediction. Given the speed and reliability of analytical results, there is a clear opportunity to exploit their use in history matching studies.

There has been much work in the literature regarding analytical approaches, particularly for well testing applications, but also from a full field reservoir standpoint, where multiple wells and reservoir boundaries must be accounted for to forecast production over the required timescales. Algorithms for full field simulation problems based on analytical approaches have been presented in the literature for porous media with homogeneous and isotropic permeability in a variety of sources. A more complex problem involves the application of analytical approaches to full field scenarios where the reservoir has inhomogeneous and anisotropic permeability and variable geometry.

The programme of research reported in this case study has been developed in collaboration with, and supported by, Schlumberger Technology Centre, Abingdon, UK. It has involved the novel development, via matched asymptotic expansions, of a predictive analytical theory for describing the flow of oil in a thin anisotropic, inhomogeneous porous layer, with injection and extraction via line sources and sinks. This provides a tractable model for oil extraction from spatially extended reservoirs. The objective has been to provide a computationally rapid tool to enable multiple realisations of extraction strategies to be assessed and compared rapidly, with the goal of optimizing the oil well locations in a new reservoir. The user friendly computational tool that has been provided via the matched asymptotic theory developed in this case study has more than achieved this goal. Pre-existing, fully numerical models require the order of days to obtain a single realisation, and are also sensitive to parameter changes, and therefore model modification. However, the computations associated with the theory developed here provide an accurate realisation in less than two minutes of computational time on MATLAB, and the computations are stable and robust to parameter changes. [redacted] (Group Leader, Schlumberger Technology Centre, Abingdon) has commented '*This new approach now makes strategy assessment for new oil reservoirs, via multiple computational realisations, followed by optimization, a significantly viable and attractive approach. Ideas from this approach have been implemented by Schlumberger in their recent efforts to build a semi-analytical Gridless Reservoir Estimation and Analysis Tool (GREAT)'* . Moreover, Schlumberger report that there is further good potential to employ this method in parallel with pre-existing numerical simulators at Schlumberger, to speed up performance, and therefore to considerably extend their viability as optimisation tools. The project has been supported by four one-day workshops in May 2006, June 2007, May 2008 and September 2009, involving the Computational Oil Recovery Research Group at Schlumberger Technology Centre, Abingdon. The workshops focussed on dissemination and discussion of the developing theory, its application and implementation, and computational methodology and efficiency. In addition D J Needham presented two research seminars at Schlumberger Technology Centre, Abingdon, relating to the description of the theory being developed and aspects of implementation of the theory. In a broader context, the research project outlined in this case study has significant implications for the worldwide petrochemical industry through the commercial services provided by Schlumberger to this industry, particularly in respect of current resource management and the necessity to optimise extraction strategies as efficiently as possible in the face of declining reserves. Associated with this activity has been the production of three detailed reports for Schlumberger:

'The unsteady flow of a weakly compressible fluid in a thin porous layer I: Two-Dimensional Theory'. (D J Needham, S Langdon) Contract Report to Schlumberger Technology Centre, Abingdon

'The unsteady flow of a weakly compressible fluid in a thin porous layer II: Three-Dimensional Theory'. (D J Needham, S Langdon) Contract Report to Schlumberger Technology Centre, Abingdon

'The unsteady flow of a weakly compressible fluid in a thin porous layer III: Three-Dimensional Computations'. (S Langdon, D J Needham, B A Samson*, J P Gilchrist*) Contract Report to Schlumberger Technology Centre, Abingdon

The work has also led to three significant publications in high quality Applied Mathematics journals.

(* Author employed at Schlumberger Technology Centre, Abingdon)

5. Sources to corroborate the impact (indicative maximum of 10 references)

Corroboration of the impact described in this case study can be obtained from Group Leader, Schlumberger Technology Centre, Schlumberger Oilfield UK PLC.