Institution: University of Birmingham

Unit of Assessment: UOA10 – Mathematical Sciences

Title of case study: Algorithms of accurate solution reconstruction on unstructured grids in computational aerodynamics: impact on aircraft design at the Boeing Company

1. Summary of the impact (indicative maximum 100 words)

This case study demonstrates the benefits achieved when the mathematical and computational aspects of a computational fluid dynamics (CFD) problem were brought together to work on real-world aerodynamic applications. While earlier insight on the solution reconstruction problem was purely based on empirical intuition, research in the School of Mathematics at the University of Birmingham by Dr Natalia Petrovskaya has resulted in the development of the necessary synthetic judgement in which the importance of accurate reconstruction on unstructured grids has been fully recognised by the CFD researchers at the Boeing Company. Boeing has confirmed that the research has led to substantial resultant improvements in their products as well as gains in engineering productivity. For instance, wing body fairing and winglets optimization for the Boeing 787 has been done by means of CFD only. Implementation of CFD in the design of their new aircraft allowed Boeing to reduce the testing time in the wind tunnel for the 787 aircraft by 30% in comparison with testing carried out for Boeing 777. Efficient use of CFD in the design of new aircrafts has helped the Boeing Company to further strengthen their core operations, improve their execution and competitiveness and leverage their international advantage.

2. Underpinning research (indicative maximum 500 words)

Dr Natalia Petrovskaya is a lecturer in the School of Mathematics at the University of Birmingham. The underpinning research is Dr Petrovskaya's body of work on a novel and efficient solution reconstruction procedure from discrete data considered on unstructured grids with arbitrary geometry.

A least-squares (LS) method is one of the most well-known approaches in solving the problem of finding the best polynomial approximation to the input data. While general accuracy estimates of the LS method are based on the assumption that all observations made to obtain LS data should provide equally precise information, data used in many practical applications is of varying quality in terms of the uncertainty of the measurement. Thus a common approach is to use weighted least-squares approximation to improve the accuracy of LS approximation. Discontinuous weighted least-squares (DWLS) approximation is a modification of a weighted LS method that is heavily used in computational aerodynamics. The method approximates a given function at each point belonging to a set of points selected over a computational grid.

One basic feature of DWLS reconstruction that stems from the nature of computational problems where the method is exploited is that a reconstruction stencil may present a highly irregular geometry. The DWLS reconstruction on irregular meshes appeared to be a challenging and difficult problem, as the method can lose accuracy to an unacceptable limit. Earlier insight into the problem, made by researchers at Boeing and NASA, attributed poor accuracy of the method on irregular grids to the impact of distant points on the results of DWLS reconstruction. However, it turned out that inverse distance weighting of stencil points was not efficient in practical aerodynamic computations, and further insight into the problem was required. The study of this problem made by Dr Petrovskaya in 2007-08 revealed that, while the inverse distance weight function has been well investigated for points that are remote in the physical space, another class of distant points (numerically distant points) may appear in the reconstruction stencil on coarse grids.

The crucial and significant research finding was to demonstrate that the numerically distant points adversely affect the accuracy of the reconstruction but they cannot be eliminated from the stencil by inverse distance weighting. As a result of the research carried out by Dr Petrovskaya it became clear that the numerically distant points have to be weighted in the data space in order to remove them from the reconstruction stencil. Dr Petrovskaya resolved this issue by suggesting a new approach that allows the measurement of distance between points in the data space. Based on this fundamental concept, a novel reconstruction algorithm has been designed and a computational code has been written for a reconstruction procedure on unstructured grids with arbitrary geometry of grid cells. As a result of Dr Petrovskaya' research the importance of the reconstruction problem has been fully acknowledged by the Boeing CFD team and that issue was taken into account and implemented while designing a new computational toolkit.

3. References to the research (indicative maximum of six references)

Research outputs in peer-reviewed journals:

1) N.B.Petrovskaya. Discontinuous Weighted Least-Squares Approximation on Irregular Grids. CMES: Computer Modeling in Engineering & Sciences, 2008, vol.32(2), pp.69-84, doi: 10.3970/cmes.2008.032.069

2) N.B.Petrovskaya. The Accuracy of Least-Squares Approximation on Highly Stretched Meshes. Int. J. Comput. Methods, 2008, vol.5(3), pp.449 - 462, doi: 10.1142/S0219876208001558

3) N.B.Petrovskaya. Quadratic Least-Squares Solution Reconstruction in a Boundary Layer Region. Commun. Numer.Meth. Engng., 2010, vol.26 (12), pp.1721-1735, doi: 10.1002/cnm.1259.

4) N.B.Petrovskaya. Data Dependent Weights in Discontinuous Weighted Least-Squares Approximation with Anisotropic Support. Calcolo, 2011, vol.48(1), pp.127-143, doi: 10.1007/s10092-010-0032-7

5) V. Wolkov [1], Ch. Hirsch, N.B.Petrovskaya. Application of a Higher Order Discontinuous Galerkin Method in Computational Aerodynamics. Mathematical Modeling of Natural Phenomena, 2011, vol.6(3) (invited issue on computational aerodynamics), pp.237-263, doi:10.1051/mmnp/20116310

Papers 1, 2 and 3 best indicate quality of research

([1]) Author in employment with the sponsoring company

Research grants:

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4. Details of the impact (indicative maximum 750 words)

This research has generated economic impact by helping improve the computational toolkit central to the application of CFD in aircraft design by engineers at The Boeing Company. These improvements are bringing financial savings for the company through reducing the amount of skilled engineering time wasted in generating unproductive results. The productivity is increased through the further improvement and validation of the computational code used in the design process. Boeing has confirmed the existence of substantial savings to the company as a result but is unable to provide financial data for commercial reasons.

CFD in aircraft design: The overall significance of CFD in the aircraft design process is now well-established. Johnson and colleagues at Boeing have said that the application of CFD has "revolutionised the process of aerodynamic design", joining the wind tunnel and flight test as primary tools, and described the resulting financial savings to their company as "tens of millions of dollars" over a twenty year period [see page 4, source 1]. CFD also provided added-value by achieving design solutions that would otherwise be unachievable, as well as shortening the design development process by reducing or eliminating the need to build successive prototypes.

Project engineers at Boeing (and elsewhere) use commercial codes to undertake CFD analyses. These codes take many years to design and validate, are applied to "live" tasks where appropriate during their development phase and are then released allowing decades of use across Boeing and a wider aerodynamics community. For instance, development work on Boeing's current "workhorse" code, TRANAIR, began in 1984 with useful results published in 1989 and on-going development in the 1990s. These codes are used extensively; Trainair was run more than 70,000 times between 1989 and 2004, with about 90 users in Boeing. The code was heavily applied in the design of aircraft such as the Boeing 777 [page 4, source 1], one of the company's best-selling products with more than 1,000 built to date.

Contribution to Boeing's new code: Boeing began the process of developing their next-generation computational code (BCFD) in 1998; as with previous codes this is already in use where appropriate within the Company, with formal release of the code and publication expected to follow in the next five years. The ultimate purpose of the new code is to allow the generation of aerodynamic data for various flow regimes about realistic complex geometries for complex geometries in a timely and affordable manner. However, the complex nature of the flows and geometries involved places substantially increased demands on the solution methodology and resources required for the design of any reliable and accurate CFD code aimed for handling complex flow.

Currently most simulations carried out at Boeing involve Reynolds-Averaged Navier-Stokes (RANS) codes. While current RANS turbulence models have been successful for analysing attached, transonic flows, whether or not these same models are applicable to complex flows with smooth surface separation is an open question. A prerequisite for answering this question is absolute confidence that the CFD codes employed reliably solve the continuous equations involved. Hence, the Boeing CFD team wanted to investigate the solver issues in detail to make sure that a correct decision about the code design would be made. It was clear that a detailed investigation of a solution reconstruction procedure on unstructured viscous grids was required. As for many discretization schemes solution reconstruction was an essential part of the scheme. Based on her earlier work as a research consultant for The Boeing Company, Dr Petrovskaya was asked by the CFD research team at Boeing to investigate the reconstruction problem in depth. The research carried out by Dr Petrovskaya has had impact in the following ways:

1. It was demonstrated that, in two and three dimensions, near singular grid node locations can cause severe problems. This is especially true for unstructured viscous grids with high aspect ratio cells and wide disparities in cell sizes and shapes, as well as for under-resolved curved boundaries. Hence based on the research by Dr Petrovskaya, the Boeing CFD team identified the solution reconstruction procedure on unstructured grids as a critical task associated with the design of a solver for computational toolkits in modern CFD (see sources 2 & 3 on page 4).

2. Cases have been documented where a higher order least-squares algorithm yielded reconstructed values two orders of magnitude larger than any values being interpolated. For grids with 30-300 million nodes it is unlikely that anomalous reconstructions would not arise and a disastrous reconstruction can feed on itself yielding worse and worse grids. Those cases helped CFD researchers at Boeing to admit that higher order solution reconstruction can be dangerous on unstructured viscous grids unless the solution latent features are resolved (sources 1 & 3, page 4). That in turn made the impact on the choice of a baseline discretization scheme used in the Boeing solver. In particular this issue has been discussed at the MTCA'09 workshop held in September 2009 in University of Birmingham (source 4).

3. The research on numerically distant points in a least-squares procedure carried out by Dr Petrovskaya revealed true nature of a large reconstruction error that appears on coarse unstructured grids. Hence Boeing researches admitted that a least-squares reconstruction procedure should be taken into account when a grid refinement algorithm is considered. The low accuracy of reconstruction may affect a solution on the initial grid and this issue must be taken into account in as well when a solution grid adaptation algorithm is designed (source 2).

Boeing's confirmation of the impact: The leaders of Boeing's CFD team have written jointly to the University corroborating the impact of Dr Petrovskaya's research in helping the company tackle important unsolved problems in 2008 that were limiting progress in advancing the applicability of CFD to its product lines. They had turned to Dr Petroskaya to address these problems because of the quality of her extensive research in the field and said that "The algorithmic problems associated with providing engineers with reliable codes to analyse such flows are unbelievably difficult. Most CFD researchers have given up and moved on to lower hanging fruit" and that Dr Petrovskaya was able to track down the source of the difficulties Boeing faced with their existing methods and provided solutions “that pointed us in the right direction” (source 5).

As a result of this input in 2008, Boeing’s subsequent and current codes have been improved and these benefits are being extended to cover further aspects of aircraft design. The current CFD toolkit (in-house computational code BCFD) has already been used in the design and aerodynamic optimization of the latest Boeing product - Boeing 787. For instance, wing body fairing and winglets optimization for the Boeing 787 has been done by means of CFD only. Implementation of CFD in the design of their new aircraft allowed Boeing to reduce the testing time in the wind tunnel for the 787 aircraft by 30% in comparison with testing carried out for Boeing 777. The company has confirmed that “The resultant improvements in our products as well as the gains in engineering productivity are substantial although quantification is again closely held.” (source 5)

5. Sources to corroborate the impact (indicative maximum of 10 references)

1. Forrester T. Johnson, Edward N. Tinoco, N. Jong Y, *Thirty years of development and application of CFD at Boeing Commercial Airplanes, Seattle,* J. Computers & Fluids 34 (2005) 1115–1151, doi:10.1016/j.compfluid.2004.06.005

2.. F. T. Johnson, D. S. Kamenetskiy, R. G. Melvin, V. Venkatakrishnan, L. B. Wigton, D. P. Young, S. R. Allmaras, J. E. Bussoletti and C. L. Hilmes. *Observations Regarding Algorithms Required for Robust CFD Codes.* Mathematical Modeling of Natural Phenomena, 2011, vol.6(3) (invited issue on computational aerodynamics), pp.2-27, doi:10.1051/mmnp/20116301

3. S. R. Allmaras, J.E. Bussoletti, C. L. Hilmes, F. T. Johnson, R.G. Melvin, E.N. Tinoco, V. Venkatakrishnan, L. B. Wigton and D. P. Young. *Algorithm Issues and Challenges Associated with the Development of Robust CFD Codes.* Variational Analysis and Aerospace Engineering, 2009, vol.33, pp.1-19, doi: 10.1007/978-0-387-95857-6_1

4. F.T.Johnson. *Algorithm Issues Associated with Extending CFD Applicability to the Full Flight Envelope.* invited lecture at the MTCA'09 workshop, 16 September 2009 (the presentation is available from forrester.johnson@boeing.com by request)

5. Corroborating statement provided jointly by Technical Fellow, The Boeing Company and Senior Technical Fellow (now retired) and currently contractor to The Boeing Company, 6th Oct 2012.