Institution: University of Bristol

Unit of Assessment: 10 - Mathematical Sciences

Title of case study: Bristol's research into multiscale methods enables more realistic modelling of real world phenomena providing benefit to industry, government and society.

1. Summary of the impact (indicative maximum 100 words)

Wavelets and multiscale methods were introduced and rapidly became popular in scientific academic communities, particularly mathematical sciences, from the mid-1980s. Wavelets are important because they permit more realistic modelling of many real-world phenomena compared to previous techniques, as well as being fast and efficient. Bristol's research into wavelets started in 1993, has flourished and continues today. Multiscale methods are increasingly employed outside academia. Examples are given here of post-2008 impact in central banking, marketing, finance, R&D in manufacturing industry and commercial software, all originating from research at Bristol. Much of the impact has been generated from the original research via software. This software includes freeware, distributed via international online repositories, and major commercial software, such as Matlab (a preeminent numerical computing environment and programming language with over one million users worldwide).

2. Underpinning research (indicative maximum 500 words)

The underpinning research consists of a body of work carried out at the University of Bristol by Bernard Silverman, FRS (Professor of Statistics) from 1993-2003 and Guy Nason (initially Lecturer, now Professor of Statistics) from 1993, supported by many graduate students and postdoctoral researchers, and funded by a variety of sources including the Engineering and Physical Sciences Research Council (EPSRC), the Royal Society, Ministry of Defence, Unilever, Wellcome and the Government Communications Headquarters (GCHQ).

Most of Bristol's early work in this area was in the development of wavelet transforms, notably the stationary wavelet transform in 1995 [1] and enabling software packages such as `wavethresh` [6], in continuous development since 1993. These developments supported a growing research effort in addressing problems in statistical curve estimation now known as wavelet shrinkage. The basic goal of wavelet shrinkage is to estimate a signal from data where the signal is contaminated by additive noise. Wavelet shrinkage operates by (i) performing the forward wavelet transform, (ii) shrinking or thresholding the coefficients and (iii) applying the inverse wavelet transform. Wavelet shrinkage is especially useful for problems where the signal has discontinuities or other irregularities that often occur in applications (e.g. edges in images) and there are deep mathematical reasons for this. These characteristics have made wavelets effective in real applications such as in image compression (for example, JPEG 2000) or fingerprint compression (the FBI fingerprint database). The Bristol Group, led by Silverman and Nason, has made several contributions: methods for correlated, irregularly spaced and network data and innovations such as the stationary wavelet transform, cross-validation for wavelets and Bayesian wavelet shrinkage methods. Below we highlight how our methods (nondecimated wavelets and complex-valued wavelets [1,4]) were used to improve core inflation modelling by the Reserve Bank of New Zealand and spatial risk measurement used by the Bank of America.

In time series analysis, stationary models, where the underlying statistical properties remain invariant with time, are ubiquitous. Unfortunately, stationary models are not appropriate for many real-world data sets and this is becoming especially apparent with the advent of the 'big data revolution'. To address this Nason, with collaborators von Sachs and Kroisandt in Germany, created the locally stationary wavelet models, [2], which have the ability to adapt to changing structure and even fast changes. Such models are capable of more realistic modelling and more accurate forecasts and have reinforced the notion that Fourier is not canonical for nonstationary modelling. We show below how these new flexible models, developed by Nason together with postgraduate student Eckley and industrialist Treloar [5], have impacted industrial texture analysis and analysis and forecasting in marketing.

3. References to the research (indicative maximum of six references)

*[1] Nason, G.P. and Silverman, B.W. (1995) The Stationary Wavelet Transform and some Statistical Applications. In Antoniadis, A. and Oppenheim, G. (eds) “Wavelets and Statistics”, *Lecture Notes in Statistics*, **103**, 281-300. DOI: 10.1007/978-1-4612-2544-7_17

[2] Nason, G.P., von Sachs, R. and Kroisandt, G. (2000) Wavelet processes and adaptive estimation of the evolutionary wavelet spectrum. *J. R. Statist. Soc. Series B*, **62**, 271-292. DOI: 10.1111/1467-9868.00231

*[3] Nason, G.P. and Sapatinas, T. (2002) Wavelet packet transfer function modelling of nonstationary time series. *Statistics and Computing*, **12**, 45-56. DOI: 10.1023/A:1013168221710

*[4] Barber, S. and Nason, G.P. (2004) Real nonparametric regression using complex wavelets. *J. R. Statist. Soc. Series B*, **66**, 927-939. DOI: 10.1111/j.1467-9868.2004.B5604.x

[5] Eckley, I.A., Nason, G.P. and Treloar, R. (2010). Locally stationary wavelet fields with application to the modeling and analysis of image texture. *J. R. Statist. Soc. Series* C, **59**, 595-616. DOI: 10.1111/j.1467-9876.2009.00721.x

[6] Nason, G.P. (1993, 2000, 2008) `wavethresh` (versions 2, 3 and 4). Available from the Comprehensive R Archive Network (software archive). URL: http://cran.rproject.org/web/packages/wavethresh/index.html

* references that best indicate the quality of the underpinning research.

4. Details of the impact (indicative maximum 750 words)

Wavelet and multiscale methods are beginning to permeate many areas of application. This case study deals with impact generated directly from research carried out in Bristol, in the areas of finance, economics, marketing and industry. The features that have made wavelets valuable in academia (speed, efficiency, performance, sparsity, theoretical guarantees) have also made them essential for many real-world applications where data can be nonstationary (time-varying statistical properties) or exhibit sharp changes (e.g. boundaries in images), and where its analysis must be fast and reliable.

In all cases described below the impact was delivered through activities such as (i) technical report publication (ii) presentation at international conferences, (iii) refereed journal publication (iv) through word of mouth transmitted by graduate students, postdocs and colleagues and, importantly, (v) through the release of high-quality free software such as the `wavethresh` [6] package, available on the Comprehensive R Archive Network (software archive). The `wavethresh` package [6] is written in the `R` language/system which is the one of the major statistical software packages and is both a statistical software environment and programming language.

Core Inflation Measures

Obtaining key measures of critical economic time series is an extremely important and challenging task. Merely collecting data to estimate inflation, for example, is expensive and time-consuming. Assessing the data and deriving estimates such as GDP or core inflation requires careful thought and delicate statistical analysis. Such economic measures have an enormous impact on decision-making at the central government level, in the wider economy and also for the general public by influencing their expectations concerning the state and future of the economy. It is, however, immensely difficult to quantify precisely the benefits of estimating inflation, much less the value of any specific method that might be involved in trying to measure or control it.

Central banks make use of state-of-the-art denoising methods to improve estimates of core inflation and other economic series. The Reserve Bank of New Zealand (RBNZ) Discussion Paper [a] highlights that New Zealand was the first world economy to introduce inflation targeting, dramatically improving its inflation performance from the worst to among the middle of the pack (Organisation for Economic Co-operation and Development 19-country average). The aim of inflation targeting is to achieve price stability. Report [a] makes it clear that "one of the main problems with measuring inflation concerns the presence of short-lived shocks that should not influence policy makers' actions." Report [a] further states: "Wavelets were specifically designed for isolating short-lived phenomena from long term trends in a signal". Report [a] then compares several wavelet shrinkage techniques including the complex-valued denoising method [4], linear wavelet shrinkage (an option in [6]) and several existing econometric methods. Report [a] also pays careful attention to issues such as use of nondecimated wavelets[1], boundary conditions and wavelets' operation as a real-time tool. Report [a]'s conclusion is that "our wavelet measure has the performance, credibility and perspicuity needed for it to be a suitable tool for central banks and other policy makers. We believe that wavelets are a very promising avenue for further research into the analysis and forecasting of economic and financial data". `wavethresh` [6] was one of the key software tools used in this work. RBNZ's wavelet measure developed up to May 2009 in [a] was actually operationally used by RBNZ to assist inflation targeting [b]. Hence, our research not only influenced RBNZ to explore multiscale methods and wavelet shrinkage methods such as those described in [4], but was used practically to feed into a mechanism to control inflation in New Zealand, [b].

Impact Through Software

Research in [1] has become influential partly by being incorporated into the MATLAB software package. MathWorks, the company that produces MATLAB, implemented both the one- and two-dimensional stationary wavelet transforms (**swt** and **swt2**) whose help pages directly reference [1]. For reasons of customer confidentiality MathWorks are unable to name their customers. However, MathWorks have directly confirmed in 2012, [e], that these transforms are being used for multiscale analysis of 1D signals in oceanographic time series and parity space analysis for sensor validation (which uses the residual signal in analysing a distributed network of sensors to determine whether all sensors are functioning properly). Also, for 2D signals they are being use for medical imaging (specifically endoscopic imaging), security inspection systems (processing of images in homeland security applications to identify harmful contents) and fault detection in manufacturing.

Increasing Marketing Accuracy

AC Nielsen is a large multinational market research company. Evidence of their utilizing multiscale methods for improving accuracy in marketing forecasts can be found in [c] and in use post-2008 [c]. The wavelet packet transfer models introduced by [3] were used to incorporate wavelet packet factors into a regression model used to forecast future sales of well-defined brands of fast moving consumer goods. The wavelet-based methods produced significantly reduced errors in future forecasting. These improvements are economically valuable for the clients of Nielsen and Nielsen itself, as well as reducing waste and energy costs from producing products at the wrong times. Document [c] also demonstrates how they used the wavelet periodogram invented in [2] to identify large variance contributions of market response time series that were not previously attributed to certain marketing independent variables, such as sales price and patterns of promotional and advertising activity, and not captured by existing algorithms.

Spatial Risk Measures and Denoising With Wavelets

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Texture Analysis in Industry

Simulating, modelling and analysing texture is an important task in many areas including manufacturing industry. Through joint work with researchers at Unilever texture analysis modelling and simulation methods were developed that were specifically applied to hair product development and new ways of analysing fabric pilling, the surface defects of textiles caused by wear, under different conditions (e.g. how hair responds to different hairsprays or how fabrics respond to different detergent/treatment regimes). The impact was delivered through an EPSRC CASE studentship and technology transfer of research methodologies [2,5] and software from Bristol into Unilever. The key theoretical development was the creation of multidimensional locally stationary wavelet processes which benefits texture modelling by (i) enabling texture features to change their nature over the spatial domain (unlike existing stationary techniques which force statistical constancy of textures) and (ii) the wavelet part permitting sharp changes of texture, which is hard to do with classical Fourier texture techniques. A key commercial advantage was that the multiscale techniques permitted a new objective measure of texture to discriminate between good or bad conditions. The Project Leader Advanced Measurement and Data Modelling, Unilever R&D, Port Sunlight [f] has commented "The work initially developed there has informed my ideas of what that particular technology can do for us and internally rewritten version of the code is still in use within the company". Much of the original code is now contained within the freeware R package `LS2W` which depends on the `wavethresh` [6] package, both on the Comprehensive R Archive Network repository. Our work resulted in a follow-on grant from Unilever for £20k that supported postdoctoral work on multiscale methods to analyse body dynamics data. Overall, Unilever supported this programme of work for five years. The research produced a trained student who subsequently worked for Shell Research for several years and then entered academia, carrying out further funded related research projects with Unilever and other companies.

5. Sources to corroborate the impact (indicative maximum of 10 references)

[a] Baqaee, David (2009) Using wavelets to measure core inflation: the case of New Zealand. Reserve Bank of New Zealand Discussion Paper Series, DP2009/05, ISSN 1177-7567. URL: http://www.rbnz.govt.nz/research/discusspapers/dp09_05.pdf
[This report corroborates how wavelets were used by RBNZ to measure core inflation.]

[b] RBNZ, *Research Manager*. Can be emailed to corroborate.
[This research manager may be contacted to corroborate the use of wavelets by Baqaee and others at RBNZ.]

[c] Personal communication from Senior Analytic Consultant, ACNielsen Analytic Consultancy Services, Emerging Markets: , (2011) and . "Increasing Marketing Accuracy: Wavelet based forecasting techniques", an "ACNielsen" publicity document presented at the ESOMAR Congress Conference, London, September 2006. (23 pages). Can be supplied on request or see URL: http://www.esomar.org/web/research_papers/Conjoint-Analysis_1426_Increasing-marketing-accuracy-br-Wavelet-based-forecasting-techniques.php.
[The publicity document corroborates that wavelet techniques for market forecasts based on [2,3] were used at ACNielsen and the personal communication corroborates that they continued to be used up until at least 2009.]

[d] *[text removed for publication]*

[e] Mathworks Inc, (2012) *Signal Processing and Communication Product Group* have supplied information on the use of the stationary wavelet transform and Mathwork's MATLAB help pages for **swt** and **swt2** functions. Can be supplied on request.
[Summary: used to corroborate Impact Through Software.]

[f] Unilever R&D Port Sunlight. *Project Leader Advanced Measurement and Data Modelling*. Letter from Unilever to confirm the use of wavelets in texture analysis.
[Summary: used to corroborate Texture Analysis in Industry.]