Institution: University of York

Unit of Assessment: 10, Mathematical Sciences

Title of case study: How far can mathematical models of war and combat be trusted?

1. Summary of the impact

What is the best way to organize firepower in war, and what weight should be placed on mathematical models? The oldest and simplest approach is dynamical-systems based and begins with Lanchester's models. Recent work has exposed some of the subtleties and limitations of these, and the dangers in the interplay between the models and organizational culture and doctrine. Above all it has been demonstrated that Lanchester's 'square law' does not apply to the use of air power. The impact is in the form of knowledge transfer: the research has been used in the professional development of serving officers at the US Naval Postgraduate School ('America's national security research university' and the world leader in the subject) and in the community of military analysis practitioners.

2. Underpinning research

Niall MacKay is an academic mathematical physicist in the Department of Mathematics, University of York, as Lecturer since 2000, and Reader since 2009. Since 2006 he has developed broader interests in applied mathematics, especially in combat modelling and its history.

The modeling of operational and larger scale warfare nowadays tends to fall somewhere between two extremes. The first is of large, computerised simulations, with many variables and parameters, which may be used for wargaming and in procurement. The second is of much simpler models in which the underlying processes can be laid bare and analyzed. This is of importance in the development of control software, but also, at its simplest, in stimulating thought about conditions of engagement and in the understanding of these that can be achieved through mathematics. It is therefore particularly important in the thinking of military analysts and in the training and development of service personnel, and this is where its impact can be found.

The oldest and simplest, 'Lanchester' models have for their main conclusion the maxim that in modern war numbers and concentration of forces are disproportionately important, other things being equal, with fighting strength varying as individual unit effectiveness multiplied by the *square* of engaged numbers (the 'square law'). This stands in contrast to the more intuitive 'linear law', in which fighting strength is simply the product of effectiveness and numbers.

The first of **MacKay's** papers on the subject was an introduction [1]. Lanchester's model is essentially a very simple dynamical system, and the term 'Lanchestrian' is often used to describe more complex dynamical-systems based warfare modelling. Such an approach was used by **MacKay** to model combat between forces of mixed types [2], providing a neat solution, based on conserved quantities, to the question 'which of my opponents should I attack first?'. This paper also resolved a Cold War-era academic dispute about the correct use of Lanchester equations for mixed forces (W. W. Kaufmann, *Nonnuclear deterrence,* in *Alliance Security: NATO and the no-first-use question*, Brookings Inst., Washington, DC, 1983, and 1987 papers by J. W. R. Lepingwell and T. F. Homer-Dixon in *International Security*).

Next came a Lanchestrian campaign analysis of the Battle of Britain [3], which demonstrated that Lanchester's insights did not apply there. The interplay of Lanchestrian thought with the development of organizational culture in the RAF, and its implications for the 'Big Wing' controversy in the form of confusion and misconceived doctrine, were explored in a paper with historian Chris Price (York St John U.) in the leading academic journal *History* [4].

Most recently, **MacKay** greatly strengthened the conclusion that Lanchester's square law does not describe air combat by conducting a combined analysis of data from the Battle of Britain and other air campaigns of WWII, the US-Japanese Pacific war and the Korean war [5]. An earlier study of the WWII and Korean data had been used by leading US airpower authority John Warden to support his Lanchestrian claim that the casualty exchange ratio depends sensitively on the force ratio–that is, that in air combat massed numbers are disproportionately effective (J. A. Warden, *The Air Campaign: planning for combat*, Brassey's: Washington, 1989). **MacKay**'s analysis showed that the full data set does not support this claim. Indeed, a claim that airpower is symmetric (between attacker and defender) and square law would be precisely wrong. Rather, to the extent that airpower departs from the linear law, it turns out to be highly asymmetric. Further work on this is under way in collaboration with airpower historian Ian Horwood, incorporating results on the Falklands, Yom Kippur, Vietnam and other campaigns.

MacKay and Price are also now working with systems biologist **A. J. Wood** (Maths/Biology) in a significant extension of their earlier work, using techniques of Markov chain Monte Carlo simulation to investigate large-scale naval battles, in the context of the pre-WW1 incorporation of geometry and calculus into naval tactics. **MacKay** is also collaborating with Profs M. Kress and K. Lin of the US Naval Postgraduate School on, respectively, modelling counterinsurgent warfare and in further work on optimal policies for the mixed-force Lanchester problem. **MacKay** and **Wood** have supervised four summer students on aspects of warfare simulation and modelling, and many undergraduate project students.

3. References to the research

[1] **N. J. MacKay**, 'Lanchester combat models', *Mathematics Today: Bulletin of the Institute for Mathematics and its Applications* **42** (2006) 170-173, arXiv:math.HO/0606300

*[2] **N. J. MacKay**, 'Lanchester models for mixed forces with semi-dynamical target allocation', *Journal of the Operational Research Society* **60** (2009) 1421-1427
[*JORS* is a respected international peer-reviewed journal for Operational Research (OR).]
DOI:10.1057/jors.2008.97

*[3] I. R. Johnson and **N. J. MacKay**, 'Lanchester models and the Battle of Britain', *Naval Research Logistics* **58** (2011) 210-222. First published online December 2008; print publication delayed for inclusion in memorial special volume for Richard E. Rosenthal.
[*NRL* is another international, high-quality, peer-reviewed general OR journal.]
DOI: 10.1002/nav.20328

*[4] **N. MacKay** and C. Price, 'Safety in Numbers: Ideas of concentration in Royal Air Force fighter defence from Lanchester to the Battle of Britain', *History* **96** (2011) 304-325.
[*History* is one of the top journals in the discipline.] DOI: 10.1111/j.1468-229X.2011.00521.x

[5] **N. J. MacKay**, 'Is air combat Lanchestrian?', *Phalanx: the Bulletin of Military Operations Research* **44** no.4 (2011) 12-14.
http://www.mors.org/userfiles/file/phalanx/mors_phalanx_dec2011_web.pdf [*Phalanx* is the professional magazine of the military OR community, and 'presents a cross-section of important current research, meetings reports, MORS news and informative oral histories'.]

[1] is a summary of old ideas with extra original material, intended for a general mathematical readership. [2,3,4] are academic research publications in peer-reviewed international journals. [5] is original research, but is intended primarily for military analysis practitioners.

4. Details of the impact

MacKay's introduction to Lanchester combat models [1] has proven useful to a number of military analysts, from the US Army, the (US) Air Force Institute of Technology and Air Combat Command [6] and the Australian Defence Science & Technology Organization [7], and in MS or PhD theses at the US Naval Postgraduate School [8], Canadian Forces College [9] and by a USAF Operations Analyst [10]. It is also used in teaching SM212 *Differential Equations*, taught 2008-13 by Prof. W. D. Joyner and subsequently Prof. R. L. Jackson, at the US Naval Academy, to a class of around 400 midshipmen, where its notation was adopted and its text heavily quoted (Prof. Jackson commented [11] that [1] 'said it much better than I could'). The mixed-forces paper [2] has been used (as 'the latest Lanchester mixed forces model') by analysts from Taiwan [12].

The Battle of Britain work [3,4] has been presented to mixed audiences from defence and industry, including serving ranking officers, at *Mathematics in Defence 2009* (by **MacKay**, at QinetiQ, Farnborough) and *Historical Analysis for Defence and Security* (by Price, at DSTL Portsdown West, 2011). More significantly, it has been used at the US Naval Postgraduate School (NPS), as follows.

The US Navy is one of the largest technological military organizations in the world, larger than the next ten navies combined. The NPS operates more widely, as 'America's national security research university', and is the world-leader in dynamic combat modelling. It has about 2500 students on 2-year master's programmes, mostly mid-career officers from the armed forces of the USA and its allies taking sabbaticals before returning to front-line service. Within the NPS, MOVES (MOdeling, Virtual Environments and Simulation) 'is the nation's institute for defense modeling and simulation ... in support of all the services and our allies.'

Prof. T. W. Lucas teaches OA/MV 4655, *Joint Combat Modeling*, for which [3] has become compulsory reading. **MacKay** visited the NPS in 2011 for collaborative work with Profs. M. Kress, R. Szechtman and M. Atkinson on the dynamics of insurgency and to give an invited guest lecture on [3,4] entitled *Safety in Numbers: Lanchester, Fuller and the Big Wing* (Thurs 28 July 2011), attendance at which was compulsory for OA/MV 4655 students. The total audience was about 100, composed of mixed faculty and students. Thus **MacKay**'s work is informing and influencing the thinking of a wide range of mid-career officers, the purpose of whose study at the NPS is precisely to develop their operational thinking. The impact here is in this knowledge transfer, via the development of officers' understanding of combat dynamics before their return to operational practice.

Most recently the historical air combat analysis of [3,4,5] was referenced in a talk [13] on *Prediction*, given by Wayne Hughes, the NPS's Professor of OR Practice, whose *Fleet Tactics* is 'said to be in every wardroom afloat' (*ORMS Today*, August 2007). This lecture was Keynote Speech at the Military Applications Society 2012 conference, the annual meeting of the military operational analysis practitioner community. Hughes called [3-5] a 'detailed recent analysis [whose conclusions are] no theoretical matter'. Brian McCue, senior analyst at the US Center for Naval Analyses and field representative at the US Fleet Forces Command (which controls around 200 ships and 1000 aircraft), has described the results of [5] as 'a severe strike against the operational utility of [Lanchestrian] theory [for] the operational planner' [14]. Countering the belief that Lanchester's square law applies to air power will remain important for as long as planners continue to use it in support of their theses, as for example in [10].

MacKay, Price and Wood have been formally invited [15] to visit the NPS in 2013-14 to present and discuss their new work on naval simulations, and have a standing invitation from Profs Kress and Lucas to present their work in the Naval Postgraduate School Combat Modeling and Advanced Combat Modeling courses.

Finally, **MacKay** has presented his warfare modelling work widely in public engagement, giving

presentations about the Battle of Britain research for the Further Mathematics Support Programme, to the National Mathematics Teachers' Summer School (of about 30 school mathematics teachers from across the UK), and locally at various schools, societies and open days. The work has also been featured in *New Scientist* [16]. Most recently **MacKay** and **Wood**, together with historians Chris Price and Ian Horwood, organized a half-day event (25-6-2013), with lectures and wargames/simulations, on mathematics in First World War naval tactics and the Battle of Jutland as part of the *York Festival of Ideas* [17].

5. Sources to corroborate the impact

[6] Maj M. J. Artelli, USAF, *Modeling and analysis of resolve and morale for the 'long war'*, Air Force Institute Of Technology dissertation AFIT / DS / ENS / 07-02; M. J. Artelli and R. F. Deckro, 'Modeling the Lanchester Laws with System Dynamics', *Journal of Defense Modeling and Simulations* 5 (2008) 1 www.au.af.mil/au/awc/awcgate/afit/artelli_modeling_resolve_morale.pdf

[7] V. Bui, L. Bui, H. Abbass, A. Bender and P. Ray, 'On the role of information networks in logistics: An evolutionary approach with military scenarios', *Evolutionary Computation 2009*, 598-605 doi:10.1109/CEC.2009.4983000

[8] Lt C. M. Mahon, USN, *A littoral combat model for land-sea missile engagements*, NPS MS thesis, September 2007 www.dtic.mil/cgi-bin/GetTRDoc?AD=ADA473951

[9] LtCol R. Dundon, CAF, *Coping with the complexity of conflict*, Canadian Forces College MDS thesis, 2009. www.cfc.forces.gc.ca/259/290/295/286/dundon.pdf

[10] E. S. Gons, *Access challenges and implications for airpower in the Western Pacific*, Pardee RAND doctoral thesis 2010 http://www.rand.org/pubs/rgs_dissertations/RGSD267.html

[11] Prof. R. L. Jackson, USNA, email 3.8.2011

[12] P-L Liu, H-K Sun, Y-T You, 'Combined arms system dynamics model for modern land battle', CCIT Journal **41** (2012) 19-28

[13] Prof. Wayne P. Hughes (USN, retired), Professor of OR Practice, US NPS, Monterey. Keynote Address on *Prediction*, Military Applications Society 2012 conference, March 2012. Hughes is a MORS [Military Operations Research Society] Fellow and ex-President, and a winner of MORS' highest honor, the Vance R. Wanner Award for significant contributions to US national security.
10thsymposium.com/presentations/Tues%20am/
0900-0930%20Hughes%20PREDICTION%20for%20publication.pdf

[14] Dr Brian McCue, USN CNA, email 16.5.2012

[15] Prof. R. Dell, Head of Dept of OR, NPS. Letter of 30.10.2012.

[16] Kate Ravilious, 'Patterns of War', *New Scientist*, 31st July 2010, 35-39

[17] *Tales of South and North: Understanding the Battle of Jutland and its preludes in World War I* http://yorkfestivalofideas.com/2013/performances-and-films/battle-of-jutland/ , accessed 13-8-2013