Institution: University of Bristol

Unit of Assessment: 10 - Mathematical Sciences

Title of case study: Research-inspired outreach work boosts public interest in mathematics and transforms perceptions of mathematicians

1. Summary of the impact (indicative maximum 100 words)

Thousands of exhibition visitors, public lecture-goers, readers, school students and TV viewers have been encouraged to explore areas of number theory and mathematical physics as a result of public engagement initiatives in four countries by University of Bristol academics. Lay people's encounters with the Bristol scientists have also changed their view of mathematics, mathematicians and the nature of their work.

Audiences have been reached through the Royal Society Summer Science Exhibition in 2011, a science fair in 2012, an award-winning Japanese TV documentary made in 2009, popular lectures given between 2008 and 2013 and contributions to popular science books.

Research on quantum mechanics, chaos and the Riemann Hypothesis is very appealing to members of the general public who have an interest in popular science. Bristol research ties these areas together. Its dissemination through various media has captured public attention internationally and inspired non-mathematicians to consider the mysteries addressed by mathematical research.

2. Underpinning research (indicative maximum 500 words)

Over several decades mathematicians and physicists at Bristol have contributed significantly to the field of quantum chaos: the study of quantum properties of systems that classically display chaos. Areas of research that have grown out of this, and that are still active at Bristol today, are random matrix theory (and its application to number theory) and the study of quantum graphs. Research in these areas underpins the public engagement projects outlined below.

One strand of research underlying this case study is in the area of number theory. Conrey (Professor at Bristol since January 2005) is one of the foremost analytic number theorists in the world. Importantly for this case study, for many years he held the record for proving the most zeros of the Riemann zeta function obey the Riemann Hypothesis [1]. He is also involved, with Keating (Professor at Bristol since October 1995) and Snaith (Initially Lecturer, then Reader at Bristol since October 2000; graduate student at Bristol 1996-9), in the application of techniques from random matrix theory to fundamental questions in number theory. In 2000 Keating and Snaith, both in the Bristol mathematical physics group, authored two papers [2,3] which launched a new field of study. This work opened the door to the use of random matrix theory to investigate several long-standing and important number theoretical questions. In particular, insight into the hundred-year-old question of the moments of the Riemann zeta function and the subsequent related work of Conrey, Keating and Snaith has had a very considerable impact in the number theoretic community, as indicated by the large number of citations the main papers received and the two London Mathematical Society prizes awarded for this work.

The other strand of research deals with quantum graphs, another area of research at Bristol which, like random matrix theory, grew out of the field of quantum chaos. This work [4,5] has been led by Band (EPSRC postdoctoral fellow at Bristol between August 2010 and July 2013: EP/H028803/1 "New approaches for isospectrality and nodal domains"). A quantum graph is a graph whose edges are assigned lengths and the whole graph is equipped with a self-adjoint differential operator, by default the Laplacian. A quantum graph can also be thought of as a network of guitar strings tied to each other. When such a system vibrates it produces a spectrum of sounds. The spectrum of the Laplacian describes the pitches of those sounds. The Laplacian's eigenfunctions describe the different vibration modes of the graph.

Band investigated the possibility of having two different graphs that share the same spectrum (isospectral graphs). He proved a theorem that yields a method for producing isospectral graphs and implemented this method to produce various new examples of such graphs. He has solved open problems in this area and his work has been recognised widely.

3. References to the research (indicative maximum of six references)

[1] Bui, H.M., Conrey, J.B. and Young, M.P. (2011) More than 41% of the zeros of the zeta function are on the critical line. *Acta Arith*. 150(1): 35-64 DOI 10.4064/aa150-1-3.

*[2] Keating J.P. and Snaith, N.C. (2000) Random matrix theory and $\zeta(1/2+it)$, *Commun.*

Math. Phys., 214: 57-89 DOI: 10.1007/s002200000261
*[3] Keating, J.P. and Snaith, N.C. (2000) Random matrix theory and L-functions at s=1/2, *Commun. Math. Phys*., 214: 91-110, DOI: 10.1007/s002200000262
[4] Band, R., Sawicki, A. and Smilansky, U. (October 2010) Scattering from isospectral quantum graphs. *J. Phys. A: Math. Theor*. 43(41) DOI 10.1088/1751-8113/43/41/415201.
*[5] Band, R., Berkolaiko, G., Raz, H. and Smilansky, U. (2012) The Number of Nodal Domains on Quantum Graphs as a Stability Index of Graph Partitions. *Commun. Math. Phys*. 311(3):815-838 DOI 10.1007/s00220-011-1384-9.
*references that best indicate the quality of the underpinning research.

4. Details of the impact (indicative maximum 750 words)
In the activities described below, Band, Conrey, Keating and Snaith have used the intriguing and accessible nature of their research to encourage audiences in several countries to engage further in mathematics and to see the human side of mathematical investigation.

Royal Society Summer Science Exhibition: Quantum mechanics has a counter-intuitive feel that makes it intriguing to diverse audiences. The research on quantum graphs [4,5] was demonstrated to the public through a hands-on exhibit in the Royal Society Summer Science Exhibition, which took place between 5 and 10 July 2011. The Bristol team, consisting of postgraduate and undergraduate students from the mathematics and physics departments and led by Band, had to compete to win a place at the exhibition.

The exhibition “showcases the most exciting cutting-edge science and technology research” [c] and in 2011 attracted large and diverse audiences, with a total of 13,812 visitors. It “provides a unique opportunity for members of the public to interact with scientists” [c]. This face-to-face interaction is an effective way to demonstrate ideas and to highlight the human side of research. The Summer Science Exhibition achieved excellent media coverage in 2011, with news pieces appearing in most of the national broadsheet papers including The Financial Times, The Times, The Daily Mirror, The Sunday Express, The Daily Mail and The Independent. The exhibition was also featured in The Washington Post online.

The effect of the exhibition extended beyond those visitors who attended the event. The Summer Science Exhibition website had 54,639 Visits, from 35,993 Absolute Unique Visitors resulting in 179,909 Page Views. In particular, the page describing Bristol’s “Can you hear the shape of a graph” exhibit [d] has been viewed by 2,346 unique visitors. In addition, a New Scientist blog [e] about this research and the exhibit has been viewed by 5,300 people, and two YouTube movies of the exhibit in action have been viewed by 2,000 people [f]. These figures indicate the number of people reached by this activity even after the exhibition.

Public lectures: Audiences have been stimulated to explore mathematics through the many public talks Conrey and Snaith have given about their research, in particular [1,2,3], both to schoolchildren and in evening lectures to general audiences. Information follows about three representative talks for which audience feedback is available.

At a public lecture on his research at the International Centre for Mathematical Sciences in Edinburgh in January 2013, Conrey introduced number theory in a historical context, then brought the audience up to date with current research. Audience feedback gives evidence of stimulated interest [i]: “made me want to go out and become more informed”; “Excellent evening! I enjoyed some of the details of the early research by Gauss and others. It was also interesting how the topic is being approached by current mathematicians”; “Very interesting subject. … Have started doing some modelling/analysis on my own and will buy a book on the subject”; “Very interesting. Was understandable for people below undergraduate level (I am at school studying for standard grades). It encouraged me to read about the subject afterwards”; “I intend to read up some more on the Riemann Hypothesis.”

At an evening lecture to the general public for the British Science Association in 2010, Snaith’s talk on her research in number theory and random matrices received these comments [i]: “a very complex concept was explained very well indeed - one of the most interesting talks; it answered some questions I was asking about!”; “good lecture for an introduction to the subject - lots of questions to go and read about!”

In June 2013, Snaith visited 21 AS-level students at Strode College, Somerset, to speak about her research [2,3]. Her aim was to reveal the excitement of original mathematics and how some of the keys to progress are mathematical interactions and collaborations, as this is a side of mathematics that is not so apparent when the subject is studied at school. Comments from students indicate that this aim was successful [i]: “there is more to learn about maths than I thought”; “showed how mathematicians actually work”; “it has changed my view on the sort of problems they solve”. Also, seven students indicated that they would now be more inclined to study mathematics, with comments such as: “I didn’t realise maths went much further than Newton or Euclid”; “there are more areas of mathematics to study than I thought”. These comments demonstrate that being exposed to current mathematical research has changed students’ views on the subject and made a third of them more likely to study maths further.

The talks, of which the above are examples, inspire audiences to engage with maths both during and after the events, as well as to reach a clearer understanding of what mathematicians do. Such events have often reached large audiences: for example, Conrey’s keynote address at the 2012 opening of the "Imaginary" exhibition at Parque de las Ciencias [b] in Granada, Spain, is mentioned in five blogs and the Granada newspaper. Snaith’s talks have included speaking to 900 final-year school students in the “mathematics in action” programme in London, and she was chosen to give one of the two London Mathematical Society Popular Lectures in 2009.

Science project: Impact is achieved not only through dissemination of current research but also through enabling students to participate in research. The following example demonstrates the profound effect on the students involved. Conrey has co-authored a paper [j] on smooth numbers with two high school students. Their presentation on the subject won a Grand Prize at the Synopsis Silicon Valley Science Fair in California, which involved 130,000 high school students. One of the students wrote this about the experience: “Our research for the math project was both enthralling and rewarding. I had previously studied a number of math concepts outside school, but this was the first time I was part of developing and creating a new method for solving a problem. Experiencing this new level of learning where I could discover something entirely new was fantastic. The project has greatly increased my enthusiasm for research and has given me ideas for other math mysteries I'd like to investigate.” By participating in current research the students’ perception of mathematics changed. The second student said: “I would have to say that the work we did definitely changed my view about mathematical research. It was interesting seeing how mathematics could be applied in the world past high school. I've always enjoyed math but didn't really understand its direct applications past high school besides things like engineering and business. I really enjoyed seeing the kind of work done by mathematicians.”

TV documentary: Conrey and Keating (both from the Bristol mathematics department) and Berry (Bristol physics) were interviewed about their research on random matrix theory and number theory for a Japanese TV documentary (2009), "The Cosmic Code Breakers", subsequently dubbed into English. This work reached an international audience (and TV viewers are potentially a different audience from those who choose to attend public lectures). The work of the Bristol researchers, in particular [2,3], formed a central component of the programme. This multi-award-winning documentary was shown at a number of film festivals in 2010 and won the Pierre-Gilles de Gennes Prize at the Pariscience Festival for "spreading scientific knowledge through an original scenario". This festival was attended by 8,200 people, including 2,700 schoolchildren; 242 people attended the screening. The film also won a Silver Dragon Award at the China International Conference of Science & Education Producers, attended by over 200 professionals from the industry. In addition, it received the Grand Prix Japan Prize from the Japan Broadcasting Corporation - the top prize, entered in the continuing education category, from 360 entries - and received the following praise: "This outstanding program holds the tension, curiosity and interest of the audience. Interviews, brilliant graphic visuals and audio techniques come together in a comprehensible and thrilling narrative of mystery-solving. The dramatic story of prime numbers affected and changed the view of all of the jury - even those afraid of math". It was also nominated for a Banff World Television Festival Rockie Award, and shown at the Goethe Institut Science Film Festival, which was attended by 128,000 people in Thailand. The programme has been syndicated by American Public Television since January 2011.

Popular science books: The Japanese popular science book “Primes, zeta functions and arithmetic quantum chaos”, by Shin-ya Koyama [a], describes research in number theory and random matrix theory. The fact that Conrey, Keating and Snaith are three of the six mathematicians with large photos on the cover is an indication of how prominent their work is in this field and in the book. Their work, including [2,3], appears in it seven times and forms a substantial part of Chapter 15. This book has sold about 5,000 copies and at the end of 2011 was the best-selling popular science and technology book on Amazon Japan.

Four English language popular science books, produced around 2004 but still available and having impact, are also relevant here. In "The Music of the Primes", by Marcus du Sautoy [a], which is consistently in the top ten bestsellers on Amazon UK for History of Mathematics and Number Theory, Bristol research is described in detail nine times and plays a significant role in the story the book tells. Schoolchildren who visit the School of Mathematics often quote this book as a factor that inspired them to consider studying the subject. One reader on Amazon states (March 2010): “The book is packed with fascinating details about eminent mathematicians, their eccentricities, and sometimes madness. My maths interests are mainly in its applications, and I've tended to regard pure maths research as an intellectual game, but this book made me want to revisit pure maths.” [g]

In the popular book "The Riemann Hypothesis", by Karl Sabbagh [a], Brian Conrey is mentioned fourteen times, Keating eight times, while the whole Bristol team (Berry, Conrey, Keating and Snaith) accounts for about 450 lines of text.

In two further popular books on the Riemann Hypothesis, "Prime Obsession" by John Derbyshire [a] (ranked 22nd in number theory and 64th in history of mathematics by Amazon UK) and "Stalking the Riemann Hypothesis" by Daniel Rockmore [a], the research of the Bristol group is described some 14 times in each book. A reader reports on “Prime Obsession” (December 2008): “I have always felt that advanced pure mathematics is as worthy an art as painting or sculpture, and the great mathematicians as worthy artists as Van Gogh etc. But because of the inaccessibility of the subject matter to the layman this great art couldn't be widely-enough shared. With more books like Prime Obsession this wrong will be righted.” [h]

5. Sources to corroborate the impact (indicative maximum of 10 references)

[a] *Prime Obsession: Bernhard Riemann and the Greatest Unsolved Problem in Mathematics*, John Derbyshire, Plume Books, reprint 2004 ISBN-13**:** 978-0452285255; *Stalking the Riemann Hypothesis: the Quest to find the hidden law of prime numbers*, Daniel Rockmore, published by Random House, 2005, ISBN-13: 978-0224062534; *The Music of the Primes*, by Marcus du Sautoy, published by Harper Perennial, new edition 2004, ISBN-13: 978-1841155807; *The Riemann Hypothesis*, by Karl Sabbagh, published by Atlantic, reprint 2004, ISBN-13: 978-0374529352; *Primes, zeta functions and arithmetic quantum chaos*, by Shin-ya Koyama (in Japanese), Tokyo: Nihonhyoronsha, 2010, ISBN978-4-535-78553-3

[b] *Sources corroborating the impact of Conrey’s keynote address in Granada, Spain in 2012.* http://depfisicayquimica.blogspot.com/2012/06/brian-conrey-en-el-parque-de-las.html; http://www.ugr.es/~surfaces/imaginary; http://www.parqueciencias.com/sala-prensa/_detalle.html?uid=f6f4959a-b6dc-11e1-9d58-71c141a1f21c; http://www.granada.org/inet/wagenda.nsf/a6eb95bbac391668c1256e1600437e4a/46207c09803d1dcdc1257617003608d2!OpenDocument; http://www.rsme.es/content/view/1076/1/; http://canalugr.es/prensa-y-comunicacion/item/58011-brian-conrey-director-del-instituto-americano-de-matem%C3%A1ticas-impartir%C3%A1-una-conferencia-en-el-parque-de-las-ciencias; http://www.granadahoy.com/article/ocio/1284518/millon/dolares/para/quien/demuestre/la/hipotesis/matematica/riemann.html

[c] Royal Society report, “Summer Science Exhibition and Soirées 2011: review for exhibitors” and http://royalsociety.org/summer-science/

[d] http://royalsociety.org/summer-science/2011/hearing-shapes

[e] http://www.newscientist.com/blogs/nstv/2011/07/quantum-graphs-make-haunting-music.html

[f] http://www.youtube.com/watch?v=Yao2EmSeVyo; http://www.youtube.com/watch?v=3cPqY1-DaZk

[g] http://www.amazon.co.uk/product-reviews/1841155802/ref =cm_cr_pr_btm_link_next_2?ie=UTF8&pageNumber=2&showViewpoints=0

[h] http://www.amazon.co.uk/product-reviews/0452285259/ref= cm_cr_pr_viewpnt_sr_5?ie=UTF8&filterBy=addFiveStar&showViewpoints=0

[i] Feedback questionnaires from public lectures

[j] Conrey, J.B., Holmstrom, M.A. & McLaughlin, T.L. (2013) Smooth Neighbors. *Experimental Mathematics* **22**, 195-202. DOI: 10.1080/10586458.2013.768483