Institution: Loughborough University

Unit of Assessment: B10 Mathematical Sciences

Title of case study: Modelling oceanic internal waves to enhance marine and naval predictions and practices

1. Summary of the impact (indicative maximum 100 words)

Large-amplitude horizontally propagating internal solitary waves commonly occur in the interior of the ocean. This case study presents evidence to demonstrate the impact of research conducted by Professor Grimshaw at Loughborough University on the development and utilisation of Korteweg-de Vries (KdV) models of these waves, which has formed the paradigm for the theoretical modelling and practical prediction of these waves.

These waves are highly significant for sediment transport, continental shelf biology and interior ocean mixing, while their associated currents cause strong forces on marine platforms, underwater pipelines and submersibles, and the strong distortion of the density field has a severe impact on acoustic signalling.

The theory developed at Loughborough University has had substantial impact on the strategies developed by marine and naval engineers and scientists in dealing with these issues.

2. Underpinning research (indicative maximum 500 words)

Large-amplitude internal solitary waves are usually generated by the interaction of the barotropic tide with the shelf break, topographic sill or other prominent bottom features. This leads to the formation of an internal tide, which then deforms and evolves into a train of very large-amplitude internal waves, with associated large pycnocline displacements and strong currents.

Research on internal solitary waves in the coastal ocean is a very active area, conducted worldwide by physical oceanographers and marine engineers. Intensive observations have been obtained in several locations by ship-based experiments, supplemented by global satellite observations. Theoretical modelling has been based either on numerical simulations, or on the development of analytical models. It is in this latter area where the research at Loughborough University (LU) has made a major contribution, to the extent that the theoretical models developed by oceanographers worldwide now form the basic paradigms for interpreting and predicting the dynamics of internal solitary waves.

The special focus on large amplitude internal waves, which are common and a robust feature of the coastal ocean, and also in the atmospheric boundary layer, began with the arrival of Professor Grimshaw to Loughborough in 2000, and the subsequent establishment of his research group consisting of Dr. El (appointed 2005) and Dr. Khusnutdinova (appointed 2003), together with several post-doctoral fellows and post-graduate students, and has continued through to 2013. The group has collaborated internationally, notably with Pelinovsky (Institute of Applied Physics, Russia) and Helfrich (Woods Hole Oceanographic Institute, USA).

Research has focussed on four main areas: generation mechanisms, the structure of internal wave trains, the effect of bottom topography, the role of the earth's rotation. One of the most effective generation mechanisms is transcritical flow over topography where the basic model is the forced KdV equation, **[3.1, 3.2]**. The basic feature of the forced KdV equation is the appearance of undular bores, which are generated both upstream and downstream of the topography, linked by a quasi-steady structure over the topography. In **[3.1]** it is demonstrated, essentially for the first time, that the width and polarity of the obstacle are important parameters. Then in **[3.2]** the forced KdV model is extended to finite-amplitude, initially in the surface wave context. One of the significant current issues is the observation that in the generation region, often several internal wave modes are observed. In **[3.3]** this topic is addressed in a novel theoretical model, which is currently being implemented on a global scale, by Dr. Stephen Griffiths at Leeds University and collaborators.

The standard KdV model has only quadratic nonlinearity, whereas observed internal solitary waves often have very large amplitudes. Hence it has become common to include a cubic nonlinear term, leading to an extended KdV (Gardner) equation, which is now the standard model **[3.4]** and can be used for arbitrary density stratification and background current fields. One of the main thrusts of the Loughborough University research has been the investigation of how internal solitary waves deform, and possibly even disintegrate, as they propagate over variable topography. This involves a detailed study of the properties of the variable-coefficient extended KdV equation, and these have been applied to actual oceanic locations, to demonstrate the wide variety of outcomes **[3.4]**. In **[3.5]** this approach was extended to a study of how undular bores deform over a slope which focussed on how tsunamis deform over the continental slope.

Recently, it has been shown that the earth's background rotation has a marked effect on the long-time development of internal solitary waves, and significant new results are described in **[3.6]**. Importantly and surprisingly, it transpires that the long-time outcome is the formation of nonlinear wave packets, and this has profound implications for *inter alia* the interpretation of satellite observations.

3. References to the research (indicative maximum of six references)

3.1. Grimshaw R.H.J., Zhang D-H. and Chow K.W., *(2007),* Generation of solitary waves by transcritical flow over a step, *Journal of Fluid Mechanics, 587,* 235-254, DOI: 10.1017/S0022112007007355

3.2. El G.A., Grimshaw R.H.J. and Smyth N.F., (2009), Transcritical shallow-water flow past topography: finite-amplitude theory, *Journal of Fluid Mechanics, 640, 187-214*, DOI: 10.1017/S0022112009991315

3.3. Griffiths S.D. and Grimshaw R.H.J. (2007), Internal tide generation at the continental shelf modeled using a modal decomposition: two-dimensional results, *Journal of Physical Oceanography, 37,* 428-451, DOI: 10.1175/JPO3068.1

3.4. Grimshaw, R., Pelinovsky, E., Talipova, T. and Kurkina, A. (2010) Internal solitary waves: propagation, deformation and disintegration, *Nonlinear Processes in Geophysics*, 17, 633-649. http://www.nonlin-processes-geophys.net/17/633/2010/npg-17-633-2010.pdf

3.5. El G.A., Grimshaw R.H.J. and Kamchatnov A.M., (2007), Evolution of solitary waves and undular bores in shallow-water flows over a gradual slope with bottom friction, *Journal of Fluid Mechanics,* 585, 213-244, DOI: 10.1017/S0022112007006817

3.6. Grimshaw, R. and Helfrich, K.R. (2012). The effect of rotation on internal solitary waves, *IMA Journal of Applied Mathematics*, 77, 326-339, DOI: 10.1093/imamat/hxs024

Grants to Professor Grimshaw:

EPSRC GR/N63642/01, *Dynamics of finite-amplitude internal and inertial wav*es, Grimshaw R.H.J., 01/ 2001-01/2004, £128,521.

EPSRC EP/D003342/1,*The effect of friction on undular bores*, Grimshaw R.H.J., 01/2005-08/2005, £4,200.

EPSRC EP/C530586/1, *Nonlinear internal gravity wave beams*, 02/2005-05/2005, £5,600

EPSRC EP/I007180/1, *Interaction of oceanic vortices with steep topography*, 09/2010-11/2010, £9,794.

EPSRC, *Interaction of vortices with topography*, 2009, £15,700.

Office of Naval Research, *Generation of the Internal Tide*, 2003-5, US$180,000.

Office of Naval Research, *Internal Solitary Waves*, 2000-2, US$110,000.

Royal Society, *Large amplitude waves with trapped cores*, 2011, £4000.

Royal Society, *Nonlinear waves in coastal seas*, 2008, £8,200.

Royal Society, *Solitary waves propagating over rough topography*, 2006, £2,480.

Royal Society, *Analytical theory of frictional undular bores*, 2004, £2,360.

Leverhulme Visiting Professor (Georgi Sutyrin), 2003, £34,000.

Leverhulme Visiting Professor (Efim Pelinovsky), 2008, £67,500.

This research has originality, rigour and significance, indicated by its publication in premier refereed journals, external funding of a total of £463,605 since 2002, and the external collaborations with two world class institutes, viz. Woods Hole Oceanographic Institute, USA and Institute of Applied Physics, Russia.

4. Details of the impact (indicative maximum 750 words)

The original and rigorous research at Loughborough University by Professor Grimshaw and colleagues since 2000 has resulted in a new modelling paradigm for non-linear oceanic waves. This research has had impact not only for fundamental science, but also for the marine engineers and ship operators worldwide who are concerned with the effect of these waves on offshore structures and submersibles. This has given tools to experimentalists and numerical modellers to find theoretical interpretations of their observations and results.

The research means that the KdV model, with its various extensions and modifications, is now routinely used as the basic paradigm for the dynamical understanding of the nonlinear internal waves commonly found in the coastal oceans. The large number of keynote presentations by Grimshaw, for example **[5.1-5.3]**, provide evidence of the widespread dissemination of this research to end users and the interest the work has attracted.

This major change extends to end-users such as marine and oil industry engineers, and to the navy scientists concerned with submersibles and acoustic signalling **[5.4]**. Examples of applications of our research comes from funded experimental work from the Office of Naval Research (ONR). Professor Louis Goodman, now at the School for Marine Science at the University of Massachusetts has carried out considerable work for the ONR using the theories of Grimshaw, **[5.5]**. Oil companies, such as BP, Exxon Mobil, StatOil, have also benefitted from this work as working offshore needs the capacity to predict the occurrence of these large waves, and to assess the impact of waves on structures and pipe lines. Submersibles routinely communicate using sound signals, which are hugely distorted by internal waves. For this reason the US Navy through ONR has conducted several major ocean experiments, utilising theory first developed at Loughborough University.

The benefits to these users include the better prediction of the occurrence of large waves leading to an improved assessment of the impact of the waves on under-sea structures and pipelines; and improved communication for submersibles whose communication is largely disrupted.

5. Sources to corroborate the impact (indicative maximum of 10 references)

The following sources of corroboration can be made available at request:

5.1. Invitation to be a speaker at Pacific Institute for the Mathematical Sciences (PIMS) Workshop “Waves in the Atmosphere and Ocean”, Vancouver, 2008: http://www.math.wisc.edu/~milewski/CRG-SFU/Home.html

5.2. Invitation to be a principal lecturer at Geophysical Fluid Dynamics (GFD) Summer School at Woods Hole Oceanographic Institution, on Nonlinear Waves, 2009: http://www.whoi.edu/fileserver.do?id=52147&pt=2&p=19387

5.3. Invitation to be a plenary lecturer at the Third DNVA-RSE Norway-Scotland Waves Symposium, Oslo, 2013: http://www.mn.uio.no/math/english/research/groups/fluid-mechanics/events/3rd-norway-scotland-waves-symposium.html

5.4. Email from Woods Hole Oceanographic Institution, USA

5.5. The Office of Naval Research in the USA uses Grimshaw’s work: http://www.onr.navy.mil/Search.aspx?q=Grimshaw