Institution: The University of Edinburgh/Heriot-Watt University (Maxwell Institute)

Unit of Assessment: B10, Mathematical Sciences

Title of case study: Valuing complex insurance liabilities using least squares Monte Carlo

1. Summary of the impact

Research by Cathcart, McNeil (both Maxwell Institute) and Morrison (Barrie & Hibbert) during the period 2008-2012 has developed a methodology based on least squares Monte Carlo to value complex insurance liabilities and manage their risks. This methodology has been adopted by Barrie & Hibbert (B&H, part of Moody's Analytics) and has enabled the company to develop an internationally leading proposition for valuing insurance products. This has generated £2.5M in revenue since 2011, through implementation in 5 new products and use in 12 new consulting projects.

2. Underpinning research

Under the Solvency II regulatory framework, insurers face the challenge of valuing assets and liabilities in a market-consistent way and projecting these values over a one-year time horizon to make sure that they have adequate capital to remain technically solvent (i.e. the value of assets exceeds the value of liabilities with high probability). One of the greatest problems in this exercise is projecting the values of complex unit-linked liabilities, such as variable annuities. These values depend on many uncertain risk factors such as interest rates, stock market prices and policyholder behaviour; they are not generally available in a closed-form formula. The lack of a simple pricing formula means that, for any hypothetical set of future risk factor values, the market-consistent value of liabilities must be estimated using Monte Carlo simulation in an appropriate pricing model.

Least-square Monte Carlo. A naive approach to the projection problem involves simulations within simulations, or 'nested' simulations. In this approach, future scenarios for the risk factors are generated from a realistic model of the real world (often called an *economic scenario generator*) under assumptions about expected policyholder mortality and behaviour. Then, a distribution of liability values is obtained by taking each scenario in turn and computing a Monte Carlo estimate of the discounted pay-off to policyholders in a market-consistent model (often referred to as risk-neutral model). This results in a nested simulation since for every 'outer' real-world scenario a large number of 'inner' risk-neutral scenarios are used to compute the Monte Carlo estimate, and this is typically costly in terms of time and resources. The least squares Monte Carlo (LSMC) approach drastically reduces the computational effort, and this is key to measuring and managing the risk on a more frequent and active basis. In the LSMC method the Monte Carlo estimates of the value of the product are computed from only a handful of risk-neutral scenarios, sometimes as few as one or two. Although these estimates are crude, they can be regressed on the real-world risk factor scenarios to obtain a polynomial function that accurately captures the relationship between risk factors and liability values. This can then be used to estimate a distribution of future values for the liabilities. Related techniques allow the calculation of sensitivities to the risk factors, which is vital information for hedging some of the financial market risk associated with the product.

New methodology. The LSMC method was originally proposed by Longstaff and Schwartz in 2001 in the context of valuing American options. The present case study lies in the area of balance sheet and capital modelling using economic scenario generators, an area for which

McNeil (Maxwell Institute, MI) and co-authors Kretzschmar and Kirchner developed a modelling framework in [1]. The research of Cathcart (MI), McNeil and Morrison (Barrie & Hibbert) demonstrated that the LSMC method could be embedded in this framework and optimized for practical use. It addressed successfully the numerous implementation issues that had to be solved in order to turn the LSMC approach into a viable production solution for unit-linked insurance products. Key advances included: the identification of the optimal balance between numbers of outer and inner scenarios when calibrating the least squares regression model; the development of metrics for the performance of the method; the choice of the order of polynomial without overfitting; the optimisation of performances for estimating extreme quantiles of the distribution of future values; the implementation of the technique for typical variable annuity structures; the computation of hedging strategies for these products by estimating the sensitivities (also known as `the Greeks'). These issues are documented in Cathcart's PhD thesis [4]. In articles written for industry journals [2-3], Cathcart and Morrison have explained the insights emerging from the thesis work for a practitioner audience. A research paper with Alexander McNeil [5] develops accurate Monte Carlo approaches to calculating variable annuity sensitivities in the form of first-order (delta) and second-order (gamma) Taylor series approximations.

Attribution. A. J. McNeil has been Maxwell Professor in the Maxwell Institute since 2006; his PhD student M.J. Cathcart graduated in 2012. The research was in collaboration with S. Morrison of Barrie & Hibbert.

3. References to the research

References marked with a * best indicate the quality of the research.

[1] Kretzschmar G., McNeil A.J. and Kirchner A., Integrated models of capital adequacy - why banks are undercapitalised, *Journal of Banking and Finance*, **34**(12), 2838-2850 (2010). http://dx.doi.org//10.1016/j.jbankfin.2010.02.028

[2] Cathcart M.J. and Morrison S., Variable annuity economic capital: the least squares Monte-Carlo approach, *Life and Pensions*, 36-40 (2009). http://www.maths.ed.ac.uk/~mthdat25/insurance/Variable-annuity-economic-capital-the-least-squares-Monte-Carlo-approach

[3] Cathcart M.J. and Morrison S., Least squares Monte Carlo simulation, *Contingencies*, **23**(2), 46-52 (2011). http://www.contingenciesonline.com/contingenciesonline/20110304?pg=55#pg49

[4]* Cathcart M.J., *Monte Carlo simulation approaches to the valuation and risk management of unit-linked insurance products with guarantees*, Doctoral thesis, Heriot-Watt University (2012). http://www.maths.ed.ac.uk/~mthdat25/insurance/Monte-Carlo-simulation-approaches-to-the-valuation-and-risk-management-of-unit-linked-insurance-products-with-guarantees

[5]* Cathcart M.J., McNeil A.J. and Morrison S., Calculating variable annuity liability 'Greeks' using Monte Carlo simulation, submitted to the *ASTIN Bulletin* (2013). http://www.maths.ed.ac.uk/~mthdat25/insurance/Calculating-variable-annuity-liability-Greeks-using-Monte-Carlo-simulation

4. Details of the impact

The methodology developed in the research has been adopted by Barrie & Hibbert (B&H) and led to the economic impact described below. B&H, a part of Moody's Analytics, is a provider of financial risk models to the financial services sector; over $25 trillion of assets and liabilities are valued and managed with B&H support; over 60% of the insurers in the Global Fortune 500 are

B&H clients. The company has benefitted from the research in three ways.

New products and services. The research has allowed B&H to develop a LSMC solution for projecting the value of complex liabilities in a computationally efficient way. This projection is a key part of the Solvency II calculation for firms adopting an internal model approach. B&H already sell a leading economic scenario generator (ESG) and the LSMC solution uses this platform and allows them to sell a new set of services to their existing international clients and to gain new clients for their ESG.

A key part of the credibility of the LSMC solution is the fact that it is backed by extensive research and that the work carried out in Cathcart's thesis has given clear answers to the implementation issues mentioned above. In particular it provides reassurance to clients that the method produces good results and that the technical choices made by B&H are backed by extensively documented investigations.

According to a senior executive of B&H, 'The commercialisation of this research effort has been one of the most successful projects that B&H has undertaken' [7]. Since 2011 B&H has taken this research and seen commercial benefit in a number of areas:

- Supporting B&H Brand – The technical aspects of these techniques and the associated promotion and marketing have helped to further support the association of the B&H brand with technical excellence in Monte Carlo simulation.
- The work has led to 12 consulting projects generating £0.75M ($1.2M) in revenue.
- It has led to 5 new product implementations generating £0.8M ($1.3M) in revenue.
- The pipeline for the balance of 2013 foresees circa £0.95M ($1.5M) in revenue for existing products.
- Estimated revenue over the next 5 years is circa £6M ($10M) representing 12% of total B&H revenue.

Provision of research-based consultancy or training. The services provided by B&H include consultancy and training for clients. There is a demand for this as the LSMC approach is a relatively sophisticated approach that requires specialist expertise that many insurers cannot routinely access. The research project has provided an extensive set of examples and training materials and has supported 6 commercial consultancy projects.

Improved risk assessment and management**.** For firms purchasing the LSMC solution the end impact of the research is improved risk assessment. Clients and their regulators have much more confidence in the estimates of the distribution of future values of complex liabilities and the capital calculations that are based on these [7].

5. Sources to corroborate the impact

[7] A senior executive of B&H will confirm the impact of the research on B&H and its products.

Note: should links to web pages be broken, please use the website http://www.maths.ed.ac.uk/~mthdat25 to access pdf versions of the pages