Institution: Keele University

Unit of Assessment: B10 Mathematical Sciences

a. Overview

Research at Keele is organised through **multi-disciplinary Research Institutes** (RI's), which provide excellent administrative support, extend funding opportunities, help to increase impact and encourage multi-disciplinary research. This structure consolidates Keele's standing as a research-led University with a strong tradition of excellence and support in areas of acknowledged research strength. Applied Mathematics, a Group of six Professors [Chapman (CJC), Fu (YF), Healey (JJH), Kaplunov (JK), Rogerson (GAR), Shrira (VS)] and two Lecturers [Naire (SN), Prikazchikov (DP)], is a key player within the ***Environment, Physical Sciences and Applied Mathematics*** (*EPSAM*) RI, and interacts closely with the RI *for* ***Science and Technology in Medicine*** (*ISTM*). Our vitality stems from the synergy of various research streams, all centred on a common core of asymptotic approaches to fundamental Continuum Mechanics problems grounded in real world challenges. Our strength is the variety of analytical techniques, complemented where appropriate by numerical simulations. Internal cohesion is instrumental in producing highly focused world-leading research, which for ease of reference and readability may be sub-divided into the three interacting streams briefly described below.

Elasticity & Acoustics: The Group's achievements include resolving fundamental aspects of the existence and non-uniqueness of elastic surface and edge waves, taking into account the effects of topography, pre-stress, anisotropy and high-frequency structure behaviour [JK3, GAR3,4]. For elastic **surface waves**, specialised formulations incorporating vertical inhomogeneity [DP4, GAR2], and mixed boundary conditions [DP2], have been also established. The developed asymptotic methodology brings drastic simplification to aid the elucidation of near-resonant phenomena of moving sources [DP1,3]. A further major insight has been achieved by modelling moving sources and solving the outstanding issue of singular behaviour in the "superboom" region in respect of wavefront surfaces with coalescing cusps [CJC4]. Important contributions to the mathematical theory of **thin, layered and periodic structures** have been made. Wave motion in layered structures with highly contrasting material properties, or face boundary conditions, has been thoroughly examined and as a result is now fully understood [CJC1, GAR 1]. A robust procedure to construct highly accurate polynomial approximations of sophisticated transcendental dispersion relations was proposed [CJC3]. The much wider applicability than the original Rayleigh-Lamb setup includes a wide range of stability problems. Recent work on **multi-scale modelling** will play a growing role in our future strategy. A powerful high frequency homogenization procedure has been developed to explicate **micro-scale dynamics** of both continuous and discrete periodic structures [JK1,2,4].

Waves and instabilities in fluids: Major advances in understanding the mechanism of laminar-turbulent transition [JJH1-3] and jet instability [JJH4] were made. For the transition in a rotating disc boundary layer, the crucial role of the previously ignored disc edge was revealed. The **absolute instability** of ubiquitous mixing layers was found to depend strongly, and counter intuitively, on the presence of external boundaries. For swirling jets, the mechanism of breakdown caused by transient algebraic growth has been established. The powerful novel mechanism of mixing in the ocean, through inertia-gravity wave breaking, was found and analytically described [VS2]. A new exact 2.5-dimensional formulation (both Eulerian and Lagrangian) of geophysical fluid dynamics on the f-plane for the whole range of Rossby numbers has also been developed [VS1]. The first **nonlinear asymptotic model** of a free surface thin film on a porous substrate, with a transition layer at the substrate boundary, has been derived and thoroughly examined; the

criteria for the onset **of linear and weakly nonlinear instabilities** was also established [SN4]. A novel method to simplify intricate dispersion relations to elementary form has been demonstrated for internal waves in the presence of water bubbles in a mixed layer; extraordinarily high accuracy is achieved by means of suitable approximations to Bessel functions of imaginary order [CJC2]. Conceptual breakthroughs for nonlinear water waves in the ocean were made. By studying the **nonlinear stability of wave fields**, it has been shown that the hitherto neglected bottom friction can completely suppress freak waves in coastal waters [VS3]. It was also discovered that predictions of probability distributions for wind wave heights could be extracted from the available forecasts of wave spectra using just symmetries [VS4].

Biomedical modelling: The thrust of this prominent research stream is incorporation of important **biomedical phenomena** into the realm of mathematical modelling, based on Continuum Mechanics. Motivated by the phenomenon of rupture of aneurysms, formation of localised bulges in membrane tubes has been rigorously investigated analytically for the first time [YF1-4], resulting in a mathematical framework under which different mechanisms that either facilitate or inhibit aneurysm formation can be quantitatively assessed. In some other work, a range of novel mathematical models have also been successful in faithfully capturing **important physiological processes**, such as voluntary cough manoeuvres, tissue growth in bioreactors and cartilage regeneration after cell therapy [SN1-3].

The aforementioned mathematical advances have a wide range of important areas of applications, extending far beyond their original motivations. For example, the framework developed for modelling of aneurysms [YF1-4] also explains localisation phenomena, and associated stability, in a variety of **engineering contexts**. The theoretical model for flow-structure interaction, during voluntary cough and forced expiratory manoeuvers, has direct medical implications for lung patients [SN3], while the model of cartilage regeneration is used by clinicians [SN2]. The conceptually new models of random weakly nonlinear wave fields, developed for wind waves [VS4], can be extended to virtually **any type of weakly nonlinear wave**. On a more practical level, these wind wave developments could also be used for design of off-shore structures and reduction of risks for navigation. The insights into the nature of laminar-turbulent transition [JJH1-3, SN4, VS2] will benefit a wide range of applications; Engineering Groups in Stockholm and Lyon are currently carrying out numerical and experimental investigations to test the theoretical predictions made in [JJH3]. The acquired understanding of jet instability [JJH4] is already being exploited for optimising diesel engines. The high-frequency homogenisation procedure [JK1,2,4] will have a major impact on both **Photonic Crystals** and the rapidly developing area of **Meta-materials**. The understanding gained from solving moving source problems [CJC4, DP1,3] will aid the modelling of high-speed trains and supersonic aircraft. Analysis of elastic waves and localisation [CJC1-3, JK3, DP2,4, GAR1-4] provides new opportunities for, and new insights into**, Non-destructive Evaluation**.

b. Research strategy

As described in RAE 2008, the broad research strategy is to produce world-leading research addressing a diverse range of challenging real-world problems. A measure of success was our 5th place for world-leading outputs in RAE 2008. Owing to well-established research quality and ability to underpin much of Keele's growing interdisciplinary research, especially Biomedical Engineering and Geophysics, the Group plays a fundamental role in **Keele's Research Strategy**. This has resulted in considerable long-standing university support and significant financial investment; evidenced by new strategic appointments, particularly low teaching loads, the purchase of a new supercomputer and internal promotions to Personal Chairs. This high level university support

enables us to retain staff and provide a **stable, sustainable and vibrant Research Group**. We believe that this support is also instrumental in the Group producing a consistent, significant and sustained research contribution.

At the Group level, our **strategy** is to build upon existing strengths and branch out into new promising areas through collaboration and networking. With two recent strategic appointments, strengthening our solids group, **Kaplunov** (previously Head of Mathematics at Brunel) and **Prikazchikov** (a former winner of the **Russian Presidential Prize for Outstanding Young Scientists**, recruited from Bauman Moscow State Technical University), we have created one of the largest UK mathematics groups in Theoretical Solid Mechanics. The whole Applied Maths Group is highly focused and produces world leading research by creating and cultivating numerous local, national and international collaborative networks. We view these networks as a major strategic asset; one which greatly enhances the breadth and depth of our research capability. Details about our collaboration and networking may be found in Section e).

A strategic decision, outlined in RAE 2008, was to establish a research presence within the new emerging direction of **Biomedical** applications. During the REF period, this has resulted in a number of substantial advances: including, establishing the mathematical foundation for the continuum-mechanical modelling of the initiation and rupture of aneurysms [YF1-4]; this involved a number of internal and external collaborators and was supported by **EPSRC, BBSRC,** the **Leverhulme Trust** and **INTAS**. Funding through a **£335K EPSRC Bridging the Gaps Project** enabled SN to lead a multi-disciplinary team to model the fluid flow in perfusion bioreactors for tissue engineering, [SN1]. With the same funding, SN, with colleagues in ISTM from the Robert Jones & Agnes Hunt Orthopaedic Hospital, Oswestry, presented a theoretical model of the Autologous Chondrocyte Implantation technique, a cell-based therapy used for the treatment of cartilage, [SN2]. [SN3] also developed a theoretical model for flow-structure interaction during voluntary cough and forced expiratory manoeuvers.

The Group has recently **significantly increased external funding**, see Section d); we also intensively promote curiosity-driven research. In addition to School and RI funding, this type of research is supported via **Personal Research Accounts**. These accounts provide a degree of financial independence and enable researchers to pursue their own medium to long term strategic aims by working on key fundamental problems, see for example [JJH1-3] and [SN3].

Future Research Strategy: We aim to build on our successes in increasing grant income, as well as both PGR and Visiting Scholar numbers. High quality PGR's and Visiting Scholars will always be of high strategic importance. Group members are mindful of the future research agenda and actively pursue new areas of significant potential. As previously mentioned, over the REF period progress has been made within the area of Bio-medical Engineering. Over the next few years areas within which potential for further development is currently being actively assessed include, *Modelling and Remote Sensing of* ***tsunamis***, ***Remote Sensing*** *of the Ocean Interior* and *Modelling of* ***Meta-Materials***. Some associated networks and funding have already been established: VS is coordinator of an EU Project *Air-Sea Interaction under Stormy and Hurricane Conditions: Physical Models and Applications to Remote Sensing*, involving partners from France, Germany and Russia. With Canadian *Natural Sciences & Engineering Research Council* support, JK spent an extended period in Alberta, developing (with Craster) a significant body of work that will underpin future developments in the Modelling of **Meta-Materials**, [JK1,2,4].

A further University Strategic Aim, fully embraced by the Group, is to increase Impact by engagement with external agencies and end-users, as detailed in our **Impact Template**. The considerable interaction we have in fact had with industry, over a number of years, is well indicated

in our **Impact Case Studies**. It is also worth remarking that CJC's expertise in acoustics (with academic colleagues from Cambridge, Manchester & Southampton, together with **industrial partners** from Thales and MOD (Portland)) is informing government decision making regarding the £25 billion project to design and build the Successor class, to replace the Vanguard class of Trident submarines.

Our **Research Strategy** is embedded within both the **School and EPSAM Strategic Plans**. These are reviewed twice a year, with resourcing requirements identified and bid for in the Annual **Business Development and Sustainability Plan**. Allocation of resources for new appointments, and for a new Super Computer, came through this well established mechanism.

c. People, including:

i. Staffing strategy and staff development

A key element of our **Staffing Strategy** is to support staff in embedding themselves within appropriate international research communities. Our success ensures that we continue to punch well above our weight in the increasingly competitive global arena, with staff developing interactions with research networks that support and sustain high quality research. Within the REF period, the eight staff have published over **100 high-quality papers** that have appeared in the leading journals for their particular field, hosted over 25 externally funded academic visitors from 13 countries, examined over 25 PhDs, and contributed to the training of around 40 PhD students at Keele and throughout Europe. This European dimension has significantly increased since RAE 2008, through two large **Marie Curie Initial Training Networks (ITNs), TANGO & LIMOUSINE,** and other collaborative projects, see Section d). Staff are in demand to present their work as plenary, keynote and invited speakers (over 200 talks around the world since 2008), collaborate in research grants and various national and international networks.

Within our **New Appointments Strategy** all new appointees must demonstrate outstanding research achievements and potential; consolidate and complement existing research strengths. These are established essential criteria in Person Specifications. **University Staffing Strategy** ensures all staff are fully supported and afforded all possible developmental opportunities. New junior staff are mentored by senior colleagues and attend a structured **Staff Development Programme**, including mandatory training before PGR supervision. An agreed research prospectus provides the background for monitoring progress, with the Annual Appraisal ensuring review, support and advice. New staff enjoy reduced teaching and administration. Keele fully supports the *Concordat to Support Career Development of Researchers* and has ***HR Excellence in Research Award*** status. **University Equality and Diversity Committee** oversees E&D, with the University Research and PGR Committees having this in their remits.

Development of researchers is a strategic aim at all levels, with formal support and monitoring through Annual Appraisal backed-up by a recently enhanced Staff Development Unit, the **Learning and Professional Development Centre**. Success in this regard is evidenced by excellent destination statistics of PGR's and contract research staff, as well as the number of promotions to personal chairs. Research staff numbers have been augmented by some very high quality international scholars funded through the **EU's Marie Curie Scheme**. Since 2008, funding has been obtained for nine Marie Curie Fellows (from Hungary, USA, India, Ireland, Italy and Russia) each typically spending between 2 and 3 years in Keele. Three new Marie Curie Fellowship applications are currently being reviewed.

University policy is to direct QR funding based on RAE success, resulting in our highly favourable staffing levels. At the Group level, to free up high quality research time, most researchers have one

teaching free semester each year. This enables YF (funded by the Chinese Central Research Agency) to spend several months a year as **Research Professor at Tianjin University**, China, VS to spend extended periods each year at Toulon University in France and JK to exploit his **Humboldt Fellowship** Status at the Technical University of Munich with extended visits to Germany. The group has benefited from two recent strategically important appointments. These appointments clearly indicate the university's strategic aim to focus resources and investment in areas of recognised research strength.

ii. Research students

PGR students are recruited nationally and internationally (**China, Russia, USA, Hungary, India, Sri Lanka** etc.). Our excellent PGR's have produced some high-quality research; see [VS2], [YF3] & [GAR3]. In keeping with Research Council and University expectations, a first class supervisory and training environment is provided. Supervisors are approved by the University PGR Committee, with mandatory Staff Development Courses and joint supervision for first-time supervisors. Students enjoy excellent research facilities and infrastructure, including individual networked PCs, access to our own Supercomputer and licensed software. **Progress and welfare** are monitored within clear University guidelines, overseen by the PGR Director. Regular formal reviews, by researchers other than the supervisor, take place. Attendance of regular Research Seminars broadens their horizons. A well-established element of PGR training is the MAGIC consortium of around 20 Maths Departments, providing video conferencing access to over fifty advanced taught courses each year. A **generic training programme** addresses the Research Councils' agenda for key and transferable skills. In addition to internal presentations, students are encouraged (and funded) to make regular conference presentations and attend advanced courses, in the UK and overseas. Progress is reviewed twice a year. **Successful completion within 4 years** has been achieved by all students in the REF period. Staff have contributed to training around 40 PhD students throughout Europe, both informally and within **LIMOUSINE** and **TANGO**.

The number of mathematics PhD awards has increased since RAE 2008. The training and research experience within our PhD programme is enhanced by interaction with **the vibrant research community** within EPSAM (almost 60 Doctoral awards over the REF period) and visiting PhD students within TANGO and LIMOUSINE. It is worth remarking that EPSAM (and mathematics) have seen a significant increase in PhD awards over REF period, with a substantial further growth in Doctoral awards envisaged for both throughout the next assessment period; we currently have eleven registered maths PhD students.

d. Income, infrastructure and facilities

The Group continues to benefit from major University **strategic investment** in research excellence, reflected in extremely favourable staffing levels. University policy creates a climate in which all research potential is maximised. Support mechanisms include (i) direct subvention to the school budget based on previous RAE ratings, resulting in reduced teaching loads and allowing more research time, and (ii) an overhead allocation scheme: all Group members hold **Personal Research Accounts** which, over and above other internal support, are used, as previously indicated, to promote personal strategic research activities and provide partial financial independence. Excellent computational facilities are provided by a dedicated UNIX network, comprising PCs and a modern Supercomputer, serviced by an IT Officer and dedicated technician. Staff requiring even more intensive computational facilities have long-standing arrangements in place to fund their use of appropriate facilities remotely; for example VS has had a long-standing arrangement through the Met. Office to access one of the world's largest **supercomputer** at the European Centre for Mid-Range Weather Forecasting. Campus wide networking and Wi-Fi allow

remote interrogation of catalogues and journal databases, complementing extensive library subscriptions covering all mainstream research interests. The RI facilitates awareness of funding opportunities, and provides **outstanding support for research grant applications**, contract negotiations, costing/pricing and overhead recovery, patenting and licensing, together with Impact advice. In addition to Seminars, a Study Group exists where staff and visitors informally present and discuss topics of mutual interest.

A key element of strategy outlined at RAE 2008 was to increase research income. The Annual Research Income per FTE, compared to that at RAE 2008, has seen a **two and half fold increase** to £36.9K. This is **over double the sector median** reported at last RAE. The total research spend has more than doubled during the REF period, to almost £1.5m; this is a 126% increase, giving an average of £295k per annum. The total spend on grants either worked on or secured within the REF period (spent through Keele or one of over 25 partners) is about **£8 Million**. These grants have been awarded by a number of funding agencies, including the EU, EPSRC, NERC, BBSRC, the Royal Society and the Leverhulme Trust, as well as funds secured direct from industry and various national overseas research funding agencies. Well in excess of £0.5 Million of income is currently available to be spent over the next REF assessment period, with three Marie Curie Fellowships, a substantial NERC application and an EU Young Investigators Grant (**total value well over £1 Million**) currently being reviewed.

Two particularly noteworthy grants are the ITNs**, LIMOUSINE** and **TANGO**. LIMOUSINE (Limit cycles of thermo-acoustic oscillators in gas turbine combustors) a 4.15 Million Euro project involved **13 European partners** and provided training for 20 PGR's and PDRF's. A second (3.73 Million Euro) Marie Curie ITN, TANGO (*Thermo-acoustic and Aero-acoustic Nonlinearities in Green combustors with Orifice structures)*, comprises of 20 industrial and academic partners,15 PGR's and PDRF's, and is coordinated in Keele. Other recent grants include **£300K** from **EPSRC** to investigate break-up of jets of fuel in combustion engines, [JJH4]. A **Marie Curie Fellow,** with VS**,** investigated rogue waves on currents, resulting in two JFM papers with four more in the pipeline. Motivated by detection of Coronary Heart Disease, a **Marie Curie Fellow,** with GAR, investigated wave propagation along arterial walls. Recently, JK obtained funding from **AMSTED Rail**, a US railway company, to use neural networks to elucidate forces within couplings.

e. Collaboration and contribution to the discipline or research base

The Group continues its service to the UK and international scientific research community by willingly providing its energy and expertise: over the REF period half the Group have served as **Research Council College Members**, with a number also acting as **Royal Society Panel Members,** as well as assisting a number of overseas research funding agencies. Most staff have significant editorial duties (including IMA Journal of Applied Mathematics, Journal of Sound and Vibration, ZAMP and Nonlinear Processes in Geophysics, as well as the Springer book Series on Nonlinear Science); all staff carry out substantial refereeing for leading international journals. Group member have also served on the Organising Committees of numerous international conferences and provided lecture course for a high number of advanced training courses for young researchers.

We believe that our various national and international networks are a significant strength. Much collaborative work has been supported by a large number of small grants, including **Royal Society Joint Project Grants, EPSRC Small Grants, LMS** and **INTAS Grants**, together with a variety of **EU** and **Research Council Network Grants**. This funding has produced a quality of research far exceeding what might be expected with such relatively small funding. With the ease of modern remote communication, together with a little internal support, these collaborations are often

extended long past the end of the grant. Moreover, some of these small collaborative projects have formed the basis for much larger projects, for example **Royal Society** and **INTAS** funding have formed the basis of successful **Marie Curie** applications. Some of the international links staff enjoy have been formally recognised. In addition to the links mentioned previously YF (China), VS (France), JK (Germany); GAR has strong links with Russia (recent awardee of the degree of ***Doctor Honoris Causa*** of *Moscow State Mining University*). The Group's numerous links often help us source exceptional international PhD students. Illustrative examples of some of our collaborative research projects are detailed below.

- [CJC4] (with **EU funding** with Ockendon (Oxford)) determined the canonical form of the *superboom* sound field in the whole region around coalescing cusps.
- Funded by the **Royal Society** [CJC3] (with Sorokin) showed that extremely accurate polynomial approximations to transcendental equations exist; a new mathematical theory of coupled waves in layered media was also developed, [CJC1].
- VS (with J.Vanneste (Edinburgh)), with £60k from EPSRC, created an extensive Wave-flow Interactions Network. This was instrumental in self-organisation of the community, greatly helped our PhD students engage with the community and resulted in a number of new ideas, for example providing the catalyst for [CJC2].
- With **INTAS** support, [GAR1] (with Mokhomodiarov and Pichugin) elucidated Neumann-Dirichlet boundary condition transition in elastic plates, resulting also in a **Marie Curie Incoming Fellowship** with application to early detection of coronary heart disease.
- With **EPSRC** support, [GAR2] (with Kiselev) extended previous studies of waves in layered elastic structures, to incorporate arbitrary vertical parametric dependence.
- With collaborative funding **from Enterprise Ireland** [VS1] (with Thomas & Voronovich) demonstrated that hitherto neglected bottom friction can suppress freak waves.
- Funded by the **Royal Society**, [VS1] (with Yakubovich) presented a wide class of exact solutions for fully nonlinear non-steady columnar motions in rotating stratified fluids.
- VS was PI of a 150k euro **INTAS** grant (Keele's share 16k Euro) concerned with sea wave dynamics and remote sensing of the sea surface, the 8 papers of output by VS included 2 *Phys Rev Letters* papers, 1 Geophysical Research Letters paper, and 1 *Journal of Fluid Mechanics* paper.
- Motivated by diagnosis of the healthy state, or otherwise, of arteries (with **INTAS** and **Royal Society** funding) [YF3] (with A. Ilichev) provided a rigorous account of solitary wave propagation in fluid-filled elastic tubes and corrected previous misconceptions.
- With Royal Society India-UK Network support, [SN4] (with Usha, IIT Madras) investigated a thin film of Newtonian fluid overlying a porous substrate. SN also ran a successful Mathematics in Medicine workshop within of an EPSRC funded Network

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