Institution: University of Kent

Unit of Assessment: 10 (Mathematical Sciences)

Title of case study: Bayesian calibration and verification of vibratory measuring devices

1. Summary of the impact

This impact case study is based on a Knowledge Transfer Partnership (KTP) between the School of Mathematics, Statistics and Actuarial Science, University of Kent and KROHNE Ltd, a world leading manufacturer of industrial measuring instruments. These precision instruments (typically flow meters and density meters) need to be calibrated accurately before being used and this is an expensive and time-consuming process.

The purpose of the KTP was to use Bayesian methodology developed by Kent statisticians to establish a **novel calibration procedure** that improves on the existing procedure by incorporating historical records from calibration of previous instruments of the same type. This reduces substantially the number of test runs needed to calibrate a new instrument and will increase capacity by up to 50%.

The impact of the KTP, which was graded as 'Outstanding', has been to change the **knowledge** and **capability** of the Company, so that they can **improve the performance** of their manufacturing process by implementing this novel calibration method. This has been achieved by adapting the underpinning Kent research to the specific context of the calibration problem, by running many calibrations to demonstrate the effectiveness of the method in practice, and by supporting the implementation of the new calibration method within the Company's core software.

Moreover, the project has **changed the Company's thinking** on fundamental science, particularly industrial mathematics. The value of historical data, and the usefulness of Bayesian methods, is now widely appreciated and training for staff in Bayesian Statistics is being introduced. Thus the project has not only changed the protocols of the Company, it has also changed their practice.

2. Underpinning research

The research in Bayesian methodology that underpins this impact case study was conducted at Kent by **Griffin** (2000–2004 and 2007–present), **Kalli** (PhD student, 2004–2007), **Walker** (2004–2013) and **X Wang** (2007–present).

Precision measuring instruments require careful calibration. For KROHNE Ltd, the current calibration procedures, which are a crucial step for the Company's production line, are time-consuming and costly. However, large amounts of historical data are available that can potentially be exploited to improve the efficiency of the calibration process. For example, for one class of meters, over 800 instruments have already been calibrated, but before the KTP project, these data were not utilised in the calibration of new meters.

The existing calibration procedure used multiple regression, based on 30 test runs under varying temperature and pressure conditions. The regression model is a second-order model with five parameters to be estimated. Bayesian methods provide a natural framework for incorporating historical data through introduction of a prior distribution and the basic aim of the KTP was to develop a Bayesian regression model for calibration.

The evidence from the historical data is that the distribution of the parameter estimates is non-normal and exhibits multimodality and skewness. This implies that a flexible family of distributions is required to model the population distribution of the meter parameters. A natural choice in a Bayesian setting is the mixture of Dirichlet process model, where we mix a multivariate normal distribution for the five parameters with a Dirichlet process random distribution function. The Kent researchers, **X Wang** and **Walker**, therefore proposed an infinite mixture model with weights attached to a set of multivariate normal distributions for the calibration process.

In this industrial application, the computational time for model fitting needs to be kept to a minimum. This led the Kent team to propose the use of simple geometric weights. References **[3.1 – 3.3]** provide the supporting theory for geometric weights. This choice of weights allows fast computation time due to the identifiability of the model. With more exotic weight structures, such as

a stick–breaking prior for the weights, the model is unidentifiable and much of the computing time is due to the Markov chain Monte Carlo algorithm visiting different parts of the model space which produce the same density.

With geometric weights it is necessary to use slice sampling ideas for dealing with the infinite nature of the model, this effectively provides a random truncation of the number of mixture components, often at a small value; this computational technique was also developed at Kent, see references **[3.4]** and **[3.5]**.

Because the new calibration procedure incorporates information from previous calibration of instruments of the same type, the number of test runs that are needed to calibrate a new meter can be reduced. The KTP Associate, overseen by **X Wang** and **Walker**, has run many tests using the proposed algorithm on data sets provided by KROHNE Ltd and the results have proved to be excellent. By applying the Bayesian pooling approach we have managed to replicate the ordinary least squares estimates of parameters for each meter using only 18 readings per meter rather than the 30 that are used in the existing multiple regression approach.

A further benefit of the Bayesian approach is improved robustness of the calibration procedure. Although the multiple regression approach works well in general, it is susceptible to occasional anomalies in the calibration data. This may result in the need to recalibrate the meter. Because the Bayesian approach 'borrows strength' from the historical data, data anomalies are less influential and this reduces the likelihood that a meter will have to be recalibrated.

Finally, the new statistical model has also highlighted an unanticipated feature which is that one of the secondary sensing devices thought to be important for calibration, namely strain gauge, is actually not required for at least two of the meter types **[5.2, 5.3]**; the elimination of this secondary device from each instrument directly reduces manufacturing cost.

A document giving full details of the proposed Bayesian procedure has been produced for the Company by the KTP Associate.

The link with KROHNE Ltd was initiated in earlier collaborative research work on mass flow measurement involving **X Wang** at Kent that resulted in a patent **[3.6]**.

3. References to the research

[3.1] Fuentes-Garcia, R., Mena, R.H. and **Walker, S.G.** (2009). A nonparametric dependent process for Bayesian regression. *Statistics and Probability Letters*, **79**, 1112-1119. doi: 10.1016/j.spl.2009.01.005

[3.2] Fuentes-Garcia, R., Mena, R.H. and **Walker, S.G.** (2010). A new Bayesian nonparametric mixture model. *Communications in Statistics*, **39**, 669-682. doi: 10.1080/03610910903580963

[3.3]* Mena, R.H., Ruggiero, M. and **Walker, S.G.** (2011). Geometric stick-breaking processes for continuous-time Bayesian nonparametric modeling. *Journal of Statistical Planning and Inference*, **141**, 3217-3230. doi: 10.1016/j.jspi.2011.04.008

[3.4]* Walker, S.G. (2007). Sampling the Dirichlet mixture model with slices. *Communications in Statistics*, **36**, 45-54. doi: 10.1080/03610910601096262

[3.5]* Kalli, M., **Griffin, J.E.** and **Walker, S.G.** (2011). Slice sampling mixture models. *Statistics and Computing*, **21**, 93-105. doi: 10.1007/s11222-009-9150-y

[3.6] Wang, X. and Hussain, Y. (2010). Method to improve mass flow measurement based on statistical analysis of signals. *This patent application was filed on January 28, 2010 in Germany (Patent DE102010006224A1), and subsequently in the USA, (Patent US2011/0184667A1).*

(References marked with a star best indicate the quality of the underpinning research.)

4. Details of the impact

The current economic climate has emphasised the importance of resource efficiency, which presents a challenge for manufacturing companies. The KTP project has developed a novel calibration procedure, based on Kent research, that exploits historical data that were already held by the Company. The benefits of the new procedure are fourfold: (i) around 50% more meters can

be calibrated in a given period of time than with the previous method, leading to increased profits and more rapid fulfilment of orders; (ii) there are separate environmental benefits, resulting from less usage of energy; (iii) for some types of meter, certain secondary measurements have been found to be unnecessary and can be eliminated; (iv) the calibration is more robust, reducing the number of instruments that need to undergo costly and time-consuming recalibration **[5.1, item 6]**. All these are confirmed by the Research and Development Manager of KROHNE Ltd, who declares that the "*new procedure based on your statistical theory reduces the overall time and also improves robustness*" and that it "*will not only benefit the Company with thousands of instruments produced more efficiently but also benefit our local economy and environment since less energy will be used in the production*" **[5.6].** These benefits will be substantial: for example the Company anticipates that gross profits will have risen by around £250,000 per annum by 2015 **[5.1, item 10]**. The collaboration with KROHNE Ltd built on initial links with the Company that were established by **X Wang**. Support for building this into a more formal collaboration, funded through a Knowledge Transfer Partnership (KTP) in 2011, was provided by the Kent Innovation and Enterprise unit.

At this stage, the **primary impact** has been to change the **knowledge** and **capability** of staff within the Company with regard to the use of historical data and specialised Bayesian methodology, so that they themselves can incorporate this **more efficient calibration method** into their manufacturing process. Previously, such methods were unknown to staff at KROHNE Ltd. The impact has been achieved through the KTP by adapting the Bayesian regression techniques developed at Kent to the specifics of the calibration problem, by running many calibrations to demonstrate the effectiveness of the method in practice, and by supporting the implementation of the new calibration method within the Company's core software. The adoption of the new production procedure based on the statistical developed by Kent statisticians is confirmed by the Research and Development Manager of KROHNE Ltd, who declares that "*this new procedure has been tested by our software engineers and it is being implemented in our core software*" **[5.6].**

The Company has been extremely positive about the benefits of the KTP throughout the project (e.g**. [5.2, 5.3, 5.5, 5.6]**) and the project has received consistently high ratings. The Company itself rated the project as "high" (the top grade) in terms of "*improved efficiency or productivity*" **[5.1, item 13]** and commented that "*The KTP **exceeded our expectations** in finding a very effective mathematical model to meet the Company's need*." **[5.1, item 18]**. The project was awarded the highest grade of "Outstanding" by the KTP Grading Panel for its achievement **[5.7]**; only around 10% of projects nationally receive this rating.

The Company's commitment to the project is evidenced by the fact that they have a software engineer working full time on the implementation. This requires a full understanding of the Bayesian methodology and the Company employed the KTP Associate for an additional five months beyond the lifetime of the KTP project to facilitate this. Part of the delivery of this knowledge transfer has been the provision of a comprehensive document detailing the statistical methods used, and associated training of the software engineers by the KTP Associate.

A **secondary impact** of the KTP has been to change the culture of KROHNE Ltd, with regard to the value of fundamental science, particularly industrial mathematics **[5.1 item 12]**. The Company now recognises the value of historical data and sees the potential for exploiting such data elsewhere in the Company to "*improve manufacturing operation and product quality*" **[5.1, item 7]**. It is also starting to address other issues in relation to historical data, such as the question of whether it is best to use all available data, or whether there may be benefits in discarding some of the oldest data.

The research undertaken by the University of Kent, with the KTP Associate, has been presented to a cross-section of staff at a seminar hosted by KROHNE Ltd **[5.4, 5.5]**. Bayesian methods were completely new to the Company, but they have been quick to realise their potential and describe the KTP as "*significant to the Company's operation for enhancing its competitiveness of precision instrumentation manufacturing*" **[5.1, item 4]**. The Company's 2013 plan includes dissemination of Bayesian statistical principles to Research & Development personnel and training of calibration operators. Thus the impact on the Company was **broad** as it affected personnel involved in research and development, implementation through software development and practical calibration. The KTP Associate has also been involved in regular discussions with KROHNE Germany.

A further example of the increased awareness of the value of statistical methods within

KROHNE Ltd is that a second software engineer is working full time on the implementation of another new statistically based procedure into the Company's software; the KTP Associate also contributed to development of this procedure.

Summary: Whilst there are various approaches to Bayesian regression, the methods used here to address the calibration problem successfully are firmly based in the nonparametric approach of the underpinning Kent research outlined in Section 2, which offers great flexibility in the choice of prior distribution; the need for this flexibility was apparent from the historical data. The impact on the Company has been to provide a new calibration procedure that utilises historical data. The new procedure reduces waste, lowers manufacturing costs and delivers more reliable products that require less re-calibration, and will lead to large increases in profits. The KTP has provided the knowledge and capability that the Company is now using to implement the procedure into its manufacturing software.

To conclude, this very successful KTP project has changed the practices and protocols of the KTP partner through transfer of knowledge and capability. The Company anticipates a gross profit increase of a quarter of a million pounds per annum in three years' time as a direct consequence of this KTP project **[5.1, item 10]**. This acknowledges the significance of this project which has already affected many departments of the Company.

5. Sources to corroborate the impact

[5.1] KTP Final Report, 6th February 2013. This provides details of the aims and objectives of the project and assessment of performance and impact by both the industrial partner and the academic partner.

[5.2] Report LMC3 from the Technical Director, KROHNE Ltd, 2nd November 2011 detailing progress and noting the seminar given by the KTP Associate. (See Contact 4.)

[5.3] Report LMC4 from the Technical Director, KROHNE Ltd, 29th February 2012, detailing progress and highlighting the "great benefits" of the approach being developed to the calibration procedures. (See Contact 4.)

[5.4] Email from the Research and Metrology Manager, KROHNE Ltd, 7th October 2011, announcing a forthcoming seminar by the KTP Associate.

[5.5] Email from the Research and Development Manager, KROHNE Ltd, 26th October 2011, highlighting the progress of the project and in particular the success of the seminar given by the KTP Associate. (See Contact 3.)

[5.6] Letter from the Research and Development Manager, KROHNE Ltd, 23rd July 2013, explaining that the new calibration procedure "*will not only benefit the Company with thousands of instruments produced more efficiently but also benefit our local economy and environment since less energy will be used in the production*." (See Contact 3.)

[5.7] Certificate of excellence, (highest grade of "Outstanding"), KTP Grading Panel.