Institution: Swansea University
Unit of Assessment: 10 – Mathematical Sciences
a. Overview

This UoA submission is composed of activities within the Department of Mathematics, one of the constituent departments of the College of Science (CoS) at Swansea, along with Biosciences, Computer Science, Physics and Geography. The CoS was established in 2011 to consolidate existing strengths in research and teaching, facilitate new, interdisciplinary research strands, and to increase the time staff can devote to research via greater administrative efficiency.

Our research can be divided into three groups (*those leaving during the census period in italics*):

Group	Members
Probability Theory (PT)	I.M.Davies, D.Finkelshtein, N.Jacob, M.Kelbert, E.Lytvynov, A.D.Neate, F.Y.Wang, J.-L.Wu, C.Yuan, *A.Truman, L.Pastur, D.Williams, A.K.Potrykus*
Algebra and Topology (AT)	E.J.Beggs, T.Brzezinski, M.D.Crossley, J.H.Giansiracusa, G.Garkusha, *F.W.Clarke*
Partial Differential Equations (PDEs)	A.Arranz-Carreno, L.Bridge, E.C.M.Crooks, N.Dancer, C.Mercuri, V.Moroz, Z.Sobol, *V.A.Liskevich (deceased), K.Zhang*

The UoA leads the Wales Institute of Mathematical and Computational Sciences (WIMCS), a £5M pan-Wales collaborative network. In particular, the UoA coordinates the Probability Theory Cluster and, jointly with Cardiff, the Analysis Cluster. The UoA also has significant involvement in the Mathematical Physics Cluster and the Computational Modelling Cluster. The UoA thus benefitted from a ca. £900k direct investment by WIMCS. Jacob and Truman were key players in winning the grant and building up the Institute.

b. Research strategy

The UoA has a clear strategy, fully supported by the CoS and the University, to host mathematical activity that has critical mass and reaches out to other disciplines to have a national and international influence. The strategy aims to: (i) **secure resources** for research; (ii) **facilitate research conduct** within the UoA; and (iii) have a **quality**-driven outlook. The implementation of our strategy is aligned with the following research processes:

i. **Securing resources**: internal peer-review of grant applications; research-incentives scheme (returning a portion of overhead income to PIs and securing dedicated time via a workload model); Departmental and College research budgets for seed-corn funding; and start-up funds for new staff.

ii. **Facilitating research conduct**: seminar series (Departmental and WIMCS clusters); dynamic research exchange via international collaboration; structured sabbatical and research-leave programme; strong mentoring of staff, particularly ECRs; vibrant cross-College interaction (with Biosciences and Engineering); centralised technical support-staff teams for IT and infrastructure; and sustainable facilities through the CoS business plan.

iii. **Optimizing research quality**: peer review of papers prior to submission to maximize quality; Departmental and College funding for conference attendance and international visits (incoming and outgoing); PR and dissemination strategy utilizing College and University news services; specialist for impact support via the EPSRC impact acceleration account (IAA), and the Department of Research and Innovation.

The post-RAE2008 objectives, as set out in the RAE5 document, were to strengthen Swansea's established research tradition in Stochastic Analysis and Applications, diversifying into the Analysis of **PDE**s (in particular non-linear PDEs), whilst continuing to strengthen the **AT** research group. WIMCS was envisaged as a major vehicle to help drive this ambition and broaden links with scientists and engineers wherever possible, whilst maintaining a firm research base in pure mathematics. Collaboration with traditional partners such as engineers, physicists and computer

scientists were to be maintained, while aspirations for new collaborative research directions, to include the Life and Environmental Sciences and Mathematics for Finance, were set out. WIMCS, with its interdisciplinary clusters, provided an additional platform for collaboration, in particular building on our existing collaboration with the College of Engineering at Swansea, thereby influencing decisions on further research directions and appointments. The collaborative philosophy of WIMCS supported the impact of our research through the filing of a patent to enhance image processing (which has attracted commercial interest) and studies on electromagnetic properties of aircraft, leading to our two impact Case Studies.

These ambitions have formed the backbone of our research development over the current census period, both enriching activity within the core discipline of mathematics, and also broadening into applied areas. For example, our **PT** group benefitted from WIMCS investment by running workshops, and most of all by running the WIMCS Probability Colloquium, which attracted scholars such as S.Albeverio, N.Bouleau, M.-F.Chen, A.Cruzeiro, M.Ledoux, B.Øksendal, M.Roeckner, K.-Th. Sturm to Swansea, as well as stimulating research collaboration with Engineering. The **PT** group was recognized for its contribution to the UK's world-leadership status in stochastic analysis in the International Review of Mathematics in the UK (IRM UK) 2010. Furthermore, a new, strong group was set up specialising in the analysis of non-linear **PDE**s. The **AT** group, unique to Wales, expanded with the addition of an EPSRC Career Acceleration Fellow (2011).

Sadly, in 2012, Professor V.A. Liskevich passed away, and our WIMCS research professor in **PDE**s departed to Nottingham in 2013. The UoA remains committed to their research fields and we have made several new appointments to this group (see staffing section). During the period of assessment, the CoS was founded to foster and stimulate multidisciplinary research and teaching. This has allowed the UoA to fulfil its ambition of broadening research activities to impact on other subject areas as set out in our 2008 aspirations, resonating with strategic directions of RCUK and EU science policy, as well as the new impact agenda, an early example being the joint supervision of research students with colleagues in Biosciences to study animal motion. Therefore it was decided to invest on a larger scale in new staff with an emphasis on applied mathematics, whilst simultaneously embracing the existing groups within the UoA (see staffing section, Ci, for details).

Major research achievements over the period include: initiating the theory of quantum singular manifolds (orbifolds) (Brzezinski); a new connection between low dimensional topology and homotopy theory (Giansiracusa); a semi-group approach to birth-death stochastic dynamics in a continuum (Finkelshtein); giving a surprising geometric interpretation of transition functions of Levy processes (Jacob); proving a quantum Mermin-Wagner theorem for a Hubbard model and Gibbs states (Kelbert); providing new, unexpected results for determinantal point processes (Lytvynov); relating log-Sobolev and Harnack inequalities to Ric and Hess (Wang); compactness results for Schrödinger semigroups with bounded-below potentials (Wang/Wu); investigating immersed structural potential methods for fluids (Arranz-Carreno); key consequences of interfacial energy terms in solid-solid phase transition models (Crooks); finite Morse index solutions of supercritical problems (Dancer); understanding the pure critical exponent problem for the p-Laplacian (Mercuri); and existence of concentration solutions to non-linear Schrödinger operators (Moroz).

Strategic plans 2014-19 are targeted at enriching the research environment of the UoA via several major initiatives:

1. We stand at the threshold of a major strategic investment by the University, in the form of our new Science and Innovation Campus. Located on a 65-acre seafront site, this development has secured £250M of investment. It will provide an unprecedented opportunity for our future research environment. Mathematics will benefit directly, occupying purpose-built accommodation that will enhance collaborative projects with major companies and agencies.
2. An expansion of staff to align with major new themes in high-performance computing, stimulated by HPC Wales in partnership with Welsh Government and Fujitsu, and by the

acquisition of a major HPC facility for Swansea associated with the Extreme Weather Event Research Centre in cooperation with the UK Met Office.

3. Cement the interdisciplinary areas between Engineering and Bioscience through the establishment of a Centre for Mathematical Modelling to invigorate interdisciplinary research, attracting research income from EPSRC, BBSRC and industry, along with CoS support for new postgraduate students and research staff, and a new Visualization Lab, sponsored by the Royal Society/Wolfson Lab Refurbishment Fund, which will enable cutting-edge work into animal motion and behaviour.

c. People, including:

I. Staffing strategy and staff development

Since RAE 2008, the UoA has completely restructured its undergraduate provision, both to improve the student experience and also to provide more flexibility for dedicated research time for staff. For example, each year around 4-5 members of staff are relieved of teaching duties for an entire semester; monitored by the CoS, research-active staff can expect a teaching-free semester every three years. This has proven successful with staff; productivity and research quality have been enhanced. Furthermore, two newly appointed full-time tutors focussing on teaching have allowed research-active staff more time to concentrate on research. The formation of the CoS has enabled many formal administrative tasks, such as recruitment, to centralised for efficiency, creating more capacity for research within departments. This additional support is invaluable given the UoA's large student cohort (380 UG and PGT students and ca. 20 PGR students).

Two distinct phases of development of the UoA have occurred during the assessment period. Before large-scale expansion was addressed, key appointments reinforcing existing groups were made: J.H. Giansiracusa, an EPSRC Career Acceleration Fellow, to strengthen the **AT**-group (2011); and N. Dancer as research professor, a Leverhulme fellow and EPSRC Visiting Researcher, to provide leadership to the **PDE** group (2013). The **PT**-group benefitted from F.-Y. Wang becoming a research professor beyond his WIMCS term, and he is now a joint appointment with Beijing Normal University, where he is the Head of the Stochastic Research Centre. Following our strategy to increase staff research time outlined above, we appointed K. Evans (2009) and I. Rodionova (2010) as full-time academic tutors, with both participating in some research activities. In addition, we appointed 4 postdocs using funding obtained from GCHQ (Heilbronn Institute).

Building on the WIMCS experience, and taking particular account of the CoS strategy to foster multi-disciplinary research, it was decided in 2012-13 to expand Mathematics in Swansea with 5 new positions, of which 4 are filled and the 5th will follow in early 2014. This resulted in the appointments of A. Arranz-Carreno, L. Bridge, D. Finkelshtein, and C. Mercuri, all applied mathematicians by training, to broaden the portfolio of interests represented in the UoA. Each new staff member is integrated into one of our existing groups, with the prospect of eventually establishing a new group in computational mathematics spinning off the **PDE** and **PT** groups. In addition the new staff joined WIMCS clusters providing immediate access to mathematical expertise across Wales and beyond.

In order to foster collaboration across subject borders, additional measures, such as joint mentoring with collaborative partners at Swansea University across discipline boundaries. The UoA and the College of Engineering have a long tradition of collaboration across the field of applied mathematics resulting in many joint publications. This interdisciplinary activity was officially recognised at the creation of WIMCS with the appointment of Prof Ken Morgan to lead the computational modelling activities at the University. The appointment of Arranz-Carreno has further cemented this collaborative programme.

Simultaneously, a new strategic partnership with Biosciences has allowed the development of biological modelling in both Departments, reinforced by 6 additional cognate appointments in Biosciences; cross-fertilisation of ideas and grant applications have already begun. A major priority is to establish a Centre for Mathematical Modelling where biologists, mathematicians, engineers

and computer scientists will work together, exploiting the facilities of the new Visualisation Lab. The appointment of Bridge aligns with this agenda.

Career-development support: Swansea University implements a development programme that encompasses all career stages. Many staff have been rewarded with promotion during the assessment period; for example, of the non-professorial staff appointed after 2007 and before 2013, more than 50% have been promoted, including two at chair level.

Sustaining the high performance of our staff is fundamental to our strategic aims and we have appropriate support programmes in place. We pride ourselves on maintaining a highly collegiate, and intellectually stimulating, research environment, reflected in a full range of activities: from a seminar series to international exchanges and visitors. Our approach to staff development owes much to our supportive and collegiate culture, which has long been a distinguishing feature of our UoA. This dovetails with our formal Performance Enabling Programme, which, in 2012, won both a *Times Higher Leadership and Management Award* and a *Universities Human Resources Excellence Award*. The programme provides all staff beyond probation with Key Performance Indicators relating to publications, grant applications and funding, and research-student supervision, informing biannual Professional Development Reviews (PDRs) with the Head of Department (Jacob). PDRs establish mutually agreed research goals and training needs. New staff are appointed a probationary supervisor and a mentor, and are encouraged to acquire the Postgraduate Certificate of Teaching in Higher Education (PGCtHE). Our probation system merges seamlessly with our Performance Enabling Programme. APECS, our training unit, allocates resources and provision in an integrated manner, responding to training needs identified in PDRs.

The University is committed to the implementation of the **2008 Concordat to Support the Career Development of Researchers**, being one of the second tranche of HEIs to be awarded the *HR Excellence in Research Award* from the European Commission, which we successfully retained in 2013. Additionally, a grass-roots collection of researchers, Swansea University Research Forum, operates a highly successful research-coaching scheme to stretch the performance of staff at all career stages. We also participate in the Women in Universities Mentoring Scheme (WUMS) for early career female academics.

Support of equalities and diversity: Of the academic staff in the UoA, 2/3 are of international origin, with nationalities including Belarus, Chinese, German, Israeli, Italian, Polish, Russian, Spanish, and Ukrainian. The UoA aims to provide an inclusive institutional culture, where differences are shared and valued, and where there is recognition of the requirement to consider and support the needs of all employees if our collective aspirations are to be met. Our commitment to diversity and equality is framed by the University's Strategic Equality Plan, which underpins a culture of inclusivity and which values diversity in all areas of activity. We run a College Equality and Diversity Forum to identify issues and deliver solutions (e.g. promoting gender balance on all committees). The University is a charter member of Athena Swan and secured the Bronze Award in 2009 and 2012. The CoS is working towards its own Bronze submission in November 2013.

During the assessment period, and with WIMCS support, the UoA hosted many visiting scholars for periods typically ca. 1 week, notably:

Visitors from the UK

C. Batty (Oxford), M. van den Berg (Bristol), G.-Q. Chen (Oxford), M. Davis (Imperial College), D. Elworthy (Warwick), V. Galaktinov (Bath), A. Kyprianou (Bath), T. Lyons (Oxford), S. Majid (Queen Mary), X. Mao (Strathclyde), M. Rushansky (Imperial College), Y. Suhov (Cambridge), J. Toland (Bath/Cambridge), B. Zegarlinski (Imperial College), T.-S. Zhang (Manchester), H. Zhao (Loughborough) and E.B. Davies (Kings College London).

Visitors from Overseas

Albeverio (Bonn), Bandle (Basel), Beghin (Roma), Bhattacharya (Caltech), Bouleau (Paris), Bozejko (Wroclaw), Buttazzo (Pisa), Caenepeel (Brussels), M.-F. Chen (Beijing), Cruzeiro

(Lisbon), Dolbeault (Paris), Fang (Dijon), Fusco (Napoli), Gannon (Alberta), Hajac (Warsaw), V.F.R. Jones (Vanderbilt), Khesin (Toronto), V. Kondratiev (Moscow), Y. Kondratiev (Bielefeld), Ledoux (Toulouse), C.Lomp (Porto), Minigione (Parma), Moerdijk (Nijmengen), Oksendal (Oslo), Olla (Paris), Panin (St. Petersburg), B. Peletier (Leiden), Ponce (Louvain), Roeckner (Bielefeld), Russo (Paris), Samko (Faro), Schaftingen (Louvain), Schapira (Paris), Schilling (Dresden),R.Schwartz (Brown), Shulman (UCSD), Smolyanov (Moscow), Sturm (Bonn), Teichmann (ETH), Thalmaier (Luxembourg), Ural'tseva (St. Petersburg), Vazquez (Madrid), Veron (Tours), Vesperi (Florence), Voigt (Dresden), Z. Jiao (Henan Normal) and Fukushima (Osaka).

c. II. Research students

In relation to its size and historical research agenda, the UoA has a high number of research students and our approach to their training was highlighted in the IRM UK 2010. Our success in attracting self-funded overseas students, together with the renewed emphasis on collaborative research with users of mathematics, will permit a further increase in the number of PGRs. In addition, within an institutional bid (calls to be expected in 2015) we expect to receive RCUK funding for research students. With CoS support the UoA has also successfully competed for University-funded scholarships in interdisciplinary areas. We work in partnership with a CoS administrative team to recruit our PGRs. In 2012, the CoS took a strategic decision to appoint a dedicated PGR recruitment officer with an international remit to grow research student numbers.

In the census period, awards include 21 PhDs, 1 MPhil and 13 MRes. The majority of our PGR-students are self-funded. The rate of PhD enrolment has risen from ca. 3 per year at the beginning of the census period to currently ca. 6. For non-UK PhD students (18 within the census period, mainly from China, the Middle east and the EU), we normally expect an entrance qualification at MSc level. Middle East students often have several years' work experience as a lecturer in their home country and, on completing their PhD usually return to their position in their home country.

With respect to PGR training, the UoA supports and adheres to the regulations of Swansea University. All PGR students are assigned a main and second supervisor who together define each student's research project, mentor the student and monitor progress throughout the student's period of candidature. PGR students have weekly contact with their supervisors, and are guided through individual reading courses in the relevant literature, before an initial research problem is assigned. Students attend generic training courses related to research through a programme initiated under the Roberts fund. They also have the opportunity to take some of our specialist undergraduate modules at level M, and are expected to participate in regular research-group seminars as well as the general Departmental colloquium. PGR students are fully integrated into the life of the UoA and have their own desk and PC, full access to ICT and library facilities (including a well-stocked Reading Room within the UoA) and associated support staff. They are all encouraged and financially supported to go to international conferences, delivering posters or oral contributions. A strict reporting system, overseen by the University, is in place, with firmly defined student probation and progression criteria involving an annual report to assess progress. The UoA has one of the best thesis completion rates in the University.

As a partner in WIMCS, we have access to EPSRC Taught Course Centres. Our students have used the Oxford-managed Centre, and **Wang** contributed a course on *Functional Inequalities and Dirichlet Forms* via this agency. However, the main mathematical training beyond individual supervision is provided by a large number of intensive ca. one-week courses given by visiting scholars, including many leaders in their fields: e.g. C Bandle (Basel), M Bozejko (Wroclaw), M Bramson (Minneapolis), R Schilling (Dresden), and JL Vazquez (Madrid). These courses include EPSRC-LMS short courses, LMS-Invited Lecturer courses, Oxford Nonlinear PDE-Centre courses, and most of all ERASMUS staff-exchange courses with Dresden, Göttingen and Wroclaw as partners. In total we had 17 courses in this assessment period open to all: 7 were in probability theory, 6 at the interface of analysis and probability theory, 3 in analysis, and 1 in algebra/topology. A typical student in probability theory or analysis attends 2 or 3 courses per year. The IRM UK 2010 explicitly applauded this approach. In addition, we support many students to attend similar courses elsewhere in the UK and abroad. We consider it important that PGR students get some teaching experience. Whilst APECS provides some training, we take care that they are mentored

in undergraduate teaching activities such as marking or small-group teaching to develop their skills further. The quality of PhD-training is recognised by the relatively large number of students who on graduation are offered post-doctoral positions or eventually get a permanent university position.

d. Income, infrastructure and facilities

As a small UoA, with a tradition of research in pure mathematics, our infrastructure needs are modest, with funding mainly used to support international collaboration (travel, inviting visiting scholars) and for a regular seminar and colloquium programme. Some support has also come via bids to the CoS Research Fund. This has enriched our research environment and allowed the development of new research interactions and projects.

During the period of assessment we benefitted from an on-going RCUK Fellowship, an EPSRC Career Acceleration Fellowship, and a Leverhulme Fellowship, as well as EPSRC and LMS grants that have supported distinguished research visitors, workshops, and joint meetings of research groups. However the impact of WIMCS has been, and remains, of supreme importance. The UoA received in total ca. £900k for WIMCS appointments, all permanent. The WIMCS income, as well as other sources (see below), allowed the UoA to run the extensive visiting scholar programme outlined in section c: 20 international workshops, 17 training courses for research students, and support for staff/PGR travel. The UoA has also benefitted from seconding a member of staff to GCHQ (Heilbronn Institute) for 3 years, bringing ca. £35k to spend on research activities in the REF period, in addition to the direct funding of PDRAs.

Swansea University has a large research library for mathematical sciences which includes access to many e-journal packages, complemented by our in-house Reading Room with ca. 2,000 volumes. In addition we have excellent IT-support, both for hardware and software, with desktop PCs for every member of staff, PDRA, and PGR student.

We are proud that a UoA of our size could spend and invest, during the REF-period, ca. £230k on PhD-students from Departmental and College funds. This is an indication of the solid financial position of the UoA and our devotion to training the next generation of mathematicians. Opening the UoA to applied avenues, in particular by taking into account priority areas of European and UK funding councils, will undoubtedly lead to more research income and, as a consequence, more research assistants, research fellows, and PhD students.

e. Collaboration and contribution to the discipline or research base

Our staff have strong collaborative partnerships with many colleagues and institutions across the world and frequently contribute to events run by international research centres such as: Heilbronn Institute, ICMS, INI, Banach Centre, Banff Institute, Chern Institute, Hausdorff Centre, IHES, MSRI Berkeley, Steklov Institute, or ZIF (Bielefeld). This collaborative ethos is fully supported by our international staff complement and the recognition bestowed on our colleagues, as listed below.

Notable Awards and Distinctions:

- **Dancer** was awarded the Hennan Medal of the Australian Academy of Science in 2009, and was a Leverhulme Fellow in 2010; He was also invited to lecture at ICM2010.
- **Finkelshtein** was awarded the Ukraine President Prize for Young Scientists in 2009, and the Ukraine President Fellowship for Young Scientists 2011- 2013.
- **Wang** was awarded, in 2009, the First Class Natural Science Prize from the Chinese Ministry of Education. He was ranked 5 across all subject areas, and was the only mathematician to receive the prize.
- **Giansiracusa** was awarded an EPSRC Career Advancement Fellowship in 2011.

Invitations

- Dancer: ICM 2010.
- Beggs: K-Theory, C*-Algebras and Topology, Chern Institute 2009.
- Brzezinski: Plenary speaker 1st ICMS 2010, UAE, Category Theory 2009, Cape Town.
- Garkusha: one-month invitation to IHES2013.
- Giansiracusa: Operads, Deformation Theory & Grothendieck-Teichmueller groups INI (Cambridge) 2013.
- Jacob: Plenary Speaker SPA Boulder 2013, ICSAA Bonn 2012, ISI – Hanzhou 2008.
- Lytvynov: Stochastic Dynamics : Mathematical Theory and Appl., ZIF (Bielefeld) 2012.
- Mercuri: invitations to the Courant Institute (R.V.Kohn) and ETH Zurich (M. Struwe).
- Moroz: SIAM Conf. Math. Aspects of Material Science 2013, SIAM Conf. Anal. PDEs 2009.
- Wang: Plenary Speaker SPA Berlin 2009, Conf. in memory of P.Malliavin 2010.
- Wu: Stochastic Analysis, Random Fields and Applications, Ascona 2011.

Organising International meetings

- E.Crooks: Mini-Symposium EQUADIFF Loughborough 2011.
- Jacob: ICSAA Bedlewo 2012, ICM Satellite Conf. in honour of M.Fukushima, Osaka 2014.
- Lytvynov: Random Matrices, Repres. Th., and Free Probability Th., Bedlewo 2011,12,13.
- Wang: Functional Inequalities and Applications SPA Session, Osaka 2010.

Journal Editorships

- Brzezinski: International Journal of Geometric Methods in Modern Physics, International Journal Mathematics and Mathematical Sciences, Pure Mathematical Sciences.
- Crooks: Bulletin, Journal, Proceedings, & Transactions of the LMS.
- Dancer: Advances in Nonlinear Studies, Advances in Differential Equations, Communications in Applied Nonlinear Analysis.
- Lytvynov: Methods in Functional Analysis and Topology.
- Moroz: Complex Variables and Elliptic Equations, Glasgow Mathematical Journal, Proceedings of the Royal Society of Edinburgh A.
- Wang: Electronic Journal of Probability Theory, Electronic Communications in Probability Theory, Journal of Theoretical Probability Theory, Science in China.
- Yuan: Journal of Control Engineering and Technology.
- Jacob is editor of the monograph series “de Gruyter Studies in Mathematics”.

Acting as Reviewer/Advisor

- Beggs: Romanian National Research Council.
- Brzezinski: EPSRC (panel), INdAM (Italy), NSFRC (Canada), FNRS (Belgium), FNP (Poland).
- Crooks and Giansiracusa : EPSRC (panel).
- Jacob: EPSRC (College), ESF (EU), DFG (Germany), Romanian RAE panel, CINECA (Italy).
- Lytvynov: Israel Science Foundation.
- Wu: Italian Government.

Workshops in Swansea

In addition to 17 Schools, we ran 20 international workshops, often with WIMCS and/or LMS support: 4 in Algebra/Topology, 8 in Analysis, 8 in Probability Theory.