Institution: King's College London

Unit of Assessment: Department of Mathematics

Title of case study: Convex optimisation in financial risk management

1. Summary of the impact (indicative maximum 100 words)

Prof. Pennanen and collaborators have developed mathematical models and computational techniques for financial risk management. The techniques allow for quantitative analysis and optimization of financial risk management actions in an uncertain investment environment. The techniques have been used by the State Pension Fund, Ministry of Social Affairs and Health, Bank of Finland and Pension Policy Institute. The techniques have significant impact on practitioners and professional services in increasing the awareness and understanding of long-term financial risks that are difficult to quantify with more traditional techniques. Beneficiaries of the developed risk management techniques include future pensioners and tax payers.

2. Underpinning research (indicative maximum 500 words)

The developed risk management techniques are based on the simple but often neglected fact that most pricing and valuation problems in the financial industry can be treated as problems of asset-liability management which, in turn, can be treated under convex optimisation. This observation adds consistency to various approaches in financial mathematics and it simplifies many problems that are often viewed as separate. A general overview is given in reference [4] below. Many earlier models concentrate solely on pricing of financial products without explicit account of the asset management side. Accounting standards, both in banking and insurance suffer from the same problem. They are often based on so called risk-neutral valuation principles or even simpler rules of thumb that ignore the interplay between assets and liabilities. This may lead to problems when the pricing models fail, as has been seen in the past.

Traditionally, the main tools in mathematical finance have come from probability theory and stochastics but convex analysis is turning out to be equally useful. For example, general characterisations of the no-arbitrage property of a perfectly liquid market model in terms of martingale measures are largely based on separation theorems for convex sets. The optimisation perspective brings in variational and computational techniques that have been successful in more traditional fields of applied mathematics such as partial differential equations and operations research. Techniques of convex optimisation allow for significant generalisations to the classical models of perfectly liquid financial markets. We have applied convex analysis techniques to extend fundamental results in mathematical finance to markets with transaction costs and portfolio constraints that are often encountered in practical applications. New models with nonlinear illiquidity effects are developed in references [1,3,5,6] below.

Techniques of convex optimisation provide new possibilities also for financial risk management in practice beyond the techniques of stochastic analysis alone. The presence of stochastic elements in a model results in difficult, often infinite-dimensional optimisation problems that require specialised optimisation techniques. We have developed new computational techniques for such problems by combining simulation techniques with the classical Galerkin method which is widely used in numerical analysis of partial differential equations and other problems in physics and engineering. A general description of this approach can be found in [4].

Practical applications of the convex optimisation techniques in risk management require statistical and econometric models of the underlying risk factors that affect the investment and the liabilities of a financial institution. Some of the models developed by our group are reported in [2,6]. Applications of the models and associated computational techniques to the Finnish pension industry are reported in articles [A,B] in Section 5 below.

The mathematical side of the research is based on earlier work of Teemu Pennanen on mathematical optimisation. The major part of it has been produced since 2008 by Pennanen at King's College London.

The underpinning research is part of the general strategy of the Financial Mathematics research group in extending the applicability of mathematical finance. The strategy is motivated by the well documented failures of more traditional models of mathematical finance during the recent financial crises.

Key researchers

- Professor Teemu Pennanen
  - King's College London since 10/2011
- Dr John Armstrong
  - Lecturer, King's College London
- Dr Petri Hilli,
  - QSA Quantitative Solvency Analysts Ltd
- Helena Aro,
  - PhD student, Aalto University

3. References to the research (indicative maximum of six references)

1*. T. Pennanen, Convex duality in optimal investment under illiquidity, *Mathematical Programming,* (2013). DOI:10.1007/s10107-013-0721-5.

2*. H. Aro, T. Pennanen, Stochastic modeling of mortality and financial markets, *Scandinavian Actuarial Journal,* 1-27, 2012. DOI:10.1080/03461238.2012.724442.

3. T. Pennanen, A.-P. Perkkiö, Stochastic programs without duality gaps, *Mathematical Programming*, 136:91-110, 2012. DOI: 10.1007/s10107-012-0552-9.

4*. T. Pennanen, Introduction to convex optimization in financial markets, *Mathematical Programming*, 134:157–186, 2012. DOI: 10.1007/s10107-012-0573-4.

5. T. Pennanen, Dual representation of superhedging costs in illiquid markets, *Mathematics and Financial Economics*, 5:233-248, 2012. DOI: 10.1007/s11579-012-0061-x.

6. P. Malo, T. Pennanen, Reduced form modeling of limit order markets, *Quantitative Finance*, 12:1025-1036, 2012. DOI:10.1080/14697688.2011.589402.

Articles marked with an asterisk best indicate the quality of the underpinning research.

4. Details of the impact (indicative maximum 750 words)

We have applied our risk management techniques to several financial institutions in Finland including the *Ministry of Social Affairs and Health, the State Pension Fund, and the Bank of Finland*. In the UK, we are developed an Asset-Liability Management (ALM) model for the Pension Policy Institute (PPI) to study the risks associated with variable annuity pension contracts. Consultations with the Ministry as well as with the Bank of Finland were initiated by them, after they had heard about our earlier work with pension insurers in Finland. Consultation with the State Pension Fund started after development of a pilot model of the state pension liabilities. Collaboration with the PPI was initiated when we contacted them and described our earlier work with Finnish pension insurers. Our consulting for the Ministry of Social Affairs and Health, the State Pension Fund, and the Bank of Finland was done through Quantitative Solvency Analysts (QSA), which is a company founded by Pennanen and his collaborators as a spin-off of their research projects. The collaboration with the PPI is a joint project aiming at developing computational tools for quantifying long term uncertainties in the UK pensions industry.

The models and computational techniques produced for the Ministry of Social Affairs and Health address the Finnish private sector pension system as a whole. Our models build on the actual cash-flows of both the assets and liabilities and it avoids many problems (most notably pro-cyclicality) associated with traditional accounting standards. The models' outputs have led to many discoveries concerning the financial risks of the pension system. The discoveries are described in reports [A], [B] and [C] the publication of which was funded by the Ministry. These reports also include new recommendations and guidelines for the financial risk management in pension insurance companies and in their regulation. The findings were widely recognised by the pension industry and the media, see for example [E,F,G,H].

Report [D] uses our asset-liability model to study how individual income generated in the UK automatic enrolment pension system compare to an income that might be considered adequate in an uncertain investment environment. Once automatic enrolment into workplace pensions is fully implemented in 2018, it is estimated that there could be between 6 and 9 million new savers into workplace pensions. The simulation studies conducted with our model suggest several possibilities to improve the likelihood of achieving adequate retirement income [D].

The consultation provided for the State Pension Fund includes risk analysis of their annual strategic investment plan as well as a long term analysis of their funding ratio. The modeling project started in the fall of 2012, and some of the computational work is still under way. The analyses have been used by the management and the board of directors to asses financial risks of the Fund. The funding ratio is the key variable used in defining the targets of the State Pension Fund set in Finnish law. The ALM-based funding ratio is currently used in risk reporting together with traditional actuarial valuations. The analyses are based on models and computational techniques developed by Pennanen and his research team.

The consultation done for the Bank of Finland focused on the risk analysis of the bank's reserves. The work included the development of a stochastic model for the investments of the bank as well as a computational analysis of the bank's investment strategies. Our analysis covered currency risk, interest rate risk, and credit risk of the banking reserves in a dynamic multiperiod setting. The results produced by the models were used by the board of directors of the Bank in the analysis of the strategic investment plan.

5. Sources to corroborate the impact (indicative maximum of 10 references)

Individual sources:

- Managing Director, State Pension Fund (testimonial received and available on request)
- Director, Ministry of Social Affairs and Health, (testimonial received and available on request)

Reports for practitioners:

A. P. Hilli, T. Pennanen, Eläkevakuuttaminen epävarmassa sijoitusympäristössä, *Unigrafia*, 2012; this 94-page report describes the results of a research project funded by the Ministry of Social Affairs and Health (in Finnish, document available on request).
B. P. Hilli, T. Pennanen, Eläkevakuuttaminen epävarmassa sijoitusympäristössä Laskelmia työeläkkeiden rahastoinnin tehostamisesta, *Aalto University*, 2012; this 50-page report describes the results of a research project funded by the Ministry of Social Affairs and Health (in Finnish, document available on request).
C. P. Hilli, T. Pennanen, Työeläkkeiden rahastoinnin uudistamistarpeet, *Työeläkelehti*, 4, 2012; invited article in Työeläkelehti, a magazine directed at experts in the field of earnings-related pension. It is published by the Finnish Centre for Pensions and comes out five times per year (in Finnish, document available on request).
D. J. Armstrong, L. Carrera, D. Redwood, T. Pennanen, What level of pension contribution is needed to obtain an adequate retirement income, Pension Policy Institute, October 2013; full report (ISBN 978-1-906284-27-5), and press release (documents available on request). Link to KCL-mirror of PPI page for full report.

Articles in newspapers (in Finnish, documents available on request):

E. Tutkijat: Eläkelaitosten sijoitukset liian lyhytnäköisiä, *Taloussanomat*, 20/06/2012
F. Tutkijoiden hurja ehdotus: Suomeen vain yksi eläkelaitos, *Taloussanomat,20/06/2012* Link to KCL-mirror of the site.
G. Avoimuus tekisi hyvää eläkkeille, Helsingin Sanomat 28.12.2012
H. Raikas tuulahdus: Tutkijakaksikko ehdottaa mielenkiintoisia uudistuksia eläkerahoitukseen Suomen Kuvalehti 12.11.2012