Institution: University of Warwick

Unit of Assessment: B10 Mathematical Sciences

Title of case study: Mathematics in Digital Art

1. Summary of the impact

This impact is on society, culture and creativity. Series and Schleimer from Warwick's Geometry and Topology group have produced attractive visualisations and physical realisations of mathematical objects arising in their research. These have elicited a wide response from members of the public, with designs being creatively used for commercial, aesthetic and educational purposes.

Series popularised a novel form of fractal art, based on the geometry of iterated Möbius maps, in her book Indra's Pearls. This has inspired many artists working in a variety of media ranging from posters to quilts. Schleimer designs elegant yet mathematically accurate realisations of three and four dimensional figures, such as knots and related surfaces, suitable for 3D printing. These are being commercially manufactured, sold and displayed publically.

2. Underpinning research

Caroline Series (Professor at Warwick since 1992) and Saul Schleimer (Assistant Professor at Warwick 2007-11 and Associate Professor from 2011) work in low dimensional topology and geometry. Their work uses modern computing power to explore and exploit concrete visualisations of the objects they study.

Series is a leading expert on hyperbolic geometry. She developed deep mathematics behind some stunning computer graphics originally created by David Mumford (Brown) and David Wright (Oklahoma) by iterating pairs of Möbius maps acting as conformal automorphisms of the Riemann sphere. By analogy to a Julia set in complex dynamics, the limit set of the iteration is the set of accumulation points of the orbit of any point under the group generated by the pair of maps. Typically, limit sets are intricately shaped fractals. Mumford and Wright discovered that certain limit sets contain spiralling chains of circles in fascinating configurations. Working with Linda Keen (City University New York), in 1992 Series used 3-dimensional hyperbolic geometry and invented "pleating rays" to establish the mathematical framework behind these experimental discoveries [1]. This justified the Mumford-Wright results mathematically and suggested further pictures. Further work put everything into the context of contemporaneous groundbreaking research in hyperbolic geometry. [2] proves there is a unique pattern of spiralling circles for each rational number.

Starting in 1995, Series joined Mumford and Wright to write a widely accessible but mathematically detailed book (Indra's Pearls) about the mathematics and computer algorithms behind the graphics which was published in 2002 [3]. From 1992-2002 her research guided and informed the exposition and also led to further graphics which are still being explored.

Schleimer began collaborating with Henry Segerman (University of Melbourne) on 3D prints in 2010. Both are experts on 3D topology. In papers intended for a wide audience of mathematicians, engineers, and artists [4, 5] they explain the mathematical tools necessary to build a variety of geometric, topological, and combinatorial sculptures via 3D printing.

In [4] they reviewed the basic theory of stereographic projection and its relation to the geometry of the three-sphere, as well as relevant facts from the theory of the quaternions. They then constructed conformally correct 3D prints of the regular four-dimensional polytopes, including the famous 120-cell. For this they had to deal with various engineering constraints which arise, such as minimal feature size and maximal printed diameter. They also gave conformally correct prints of minimal surfaces contained in the three-sphere which were first found mathematically by Lawson (1970).

Schleimer and Segerman also investigated linkages [5]. Inspired by sculptures of Helaman Furguson and Oskar van Deventer, they invented a 3D-object they call "*Triple gear*". This is the first example of a mechanism having three interlocking gears in which any one gear is able to turn if and only if the others do, without the aid of a gearbox as found in standard assemblies. The interlocking nature of the pieces, together with their demanding tolerances, makes the triple gear

all but impossible to manufacture by traditional means. *Triple gear* answers a question often presented in graphic designs: can an odd number of gears, each in contact with the previous and the next, move? If the gears all lie in a plane, the answer is "no": any such collection is frozen and cannot move. *Triple gear* avoids this contradiction because it is not planar.

3. References to the research

1. L. Keen, and **C.Series.** Pleating Coordinates for the Maskit embedding of the Teichműller space of punctured tori, Topology 32(4) 719-749. (1993) DOI: 10.1016/0040-9383(93)90048-Z.

2. L. Keen, B. Maskit and **C.Series**. Geometric finiteness and uniqueness for Kleinian groups with circle packing limit sets. J. reine und angew. Mathematik 1993(436), 209 - 219. (1993) DOI: 10.1515/crll.1993.436.209.

3. D. Mumford, **C.Series** and D. Wright, Indra's Pearls. Cambridge Univ. Press. (2002) ISBN: 978-0-521-35253-6.

4. **S. Schleimer** and H. Segerman, Sculptures in S^3. Proceedings of Bridges 2012: Mathematics, Music, Art, Architecture, Culture, (2012) 103 - 110. Available from: http://bridgesmathart.org/2012/cdrom/proceedings/53/index.html

5. **S. Schleimer** and H. Segerman, Triple gear. Proceedings of Bridges 2013: Mathematics, Music, Art, Architecture, Culture, (2013), 353-360. Available from: http://archive.bridgesmathart.org/2013/bridges2013-353.html

4. Details of the impact (indicative maximum 750 words)

Mathematics has always informed and influenced the creative arts. Schleimer and Series' research has had impact on society, culture and creativity, with additional educational and commercial aspects. Their research has been used to create elegant objects and images which spread general awareness of mathematics, and has been reproduced and modified by the interested public.

Series' work has been disseminated through the book Indra's Pearls [3] [text removed for publication] It has been translated into Russian (2011) and Japanese (2013). There have been many favourable reviews and much positive feedback, e.g. the Indra's Pearls entry on Amazon. http://www.amazon.co.uk/product-reviews/0521352533/ref=dp_top_cm_cr_acr_txt?ie=UTF8&showViewpoints=1

The images made using [3] can be broadly categorized as fractal art. The algorithms have not been copyrighted and Google searches for terms like `fractal art Kleinian' lead to many original designs produced by professional graphics artists and enthusiastic amateurs. Due to the limitation on references we can only highlight a few examples to demonstrate the reach of the impact of [3]. All were unsolicited and acknowledge that they are based on [3]; all examples cited occurred in the assessment period 2008-13.

a) A well-known Belgian mathematical graphics artist used [3] to create prints, posters and book covers "after becoming fascinated by the graphics from the book", (see his extensive website and online galleries [7]). Highlights include a set of images for the offices of the London Mathematical Society (2011) and the exhibition poster and other exhibits in the Exposition Mathématiques et Art, Paris, 2012; also an online article explaining how to make the pictures 3-dimensional.

b) An American software developer incorporated algorithms from [3] as "Kleinian Group Orbit Traps" into his Fractal Science Kit (v1 released June 2008), thus providing an interactive programming environment for creating fractals. In addition to selling this software, he also sells fractal images via on-line galleries such as RedBubble, and displays his work on Flickr and Facebook [8].

c) A British self-employed IT professional, incorporated algorithms from [3] into his freeware fractal program Spirofractal creating publically available artwork. He says that Indra's Pearls had an

"enormous impact" on him and that "it has significantly improved my skills as a software developer"[9]. Other computer artists include an American programmer who sells posters, cards and stationery inspired from [3]; a Canadian software designer who posts video clips (2009 -2011); and an American artist incorporated algorithms into his program Swirlique on his Mystic Fractal website along with other examples found on the DeviantART gallery [9].

[3] has also been valued for its educational component. Series has given many public lectures: in the assessment period, the LMS/IMA Gresham Lecture (2010) *Indra's Pearls: Geometry and Symmetry*, (available at http://www.lms.ac.uk/events/lectures/gresham-lectures) and a similar talk at the Oxford Mathematical Institute Garden Party (2011). [3] has significantly and directly impacted the extensive popularization work of a German mathematician and computer visualisation specialist (TU Munich). He created 25 interactive applets based on [3] for a summer camp for mathematically gifted high school students. Now available on line in three languages and frequently used, these also led to enhancements of his geometry software Cinderella (2009) for schools and the general public. The applets were the initial step in the creation of his internet platform 'mathe-vital.de' for mathematical experiments for schools, peaking at 700 visits per day and winning the 2008 MedidaPrix. The work also led to an installation at the Deutsche Museum, Munich 2011(1.4M visitors/year), other art exhibitions and a prizewinning iOS app iOrnament [10].

The reach of Indra's Pearls extends to other media. It drew attention to the analogy between the iterative processes involved and eastern philosophical traditions, and images have been used as jacket designs for two books on eastern philosophy [11] Other designs based on [3] include a widely exhibited silk quilt named *Indra's Pearls* by a quilt designer (2009) [12], and an engagement ring for a lady from Arizona [12] "… a quick note to thank you all for your work on Indra's Pearls, and to share the engagement ring that my partner had made for me based on the math he learned in your book...".

Schleimer-Segerman's 3D prints are for sale in attractive format from Shapeways. [text removed for publication]

Public interest has been raised through the STL files for *Triple gear* ([text removed for publication]) and a related sculpture, *Triple helix*, available via Thingiverse [14]. With the STL file, a 3D modelling program, and a 3D printer, the design can be modified and replicated at will. Videos discussing each piece are posted on YouTube. The video on *One-half 120-cell* has been watched over 17,000 times since 9/2011 and that on *Triple gear* more than 113,000 times since 12/2012. Every time such a print is posted it generates online discussion on blogs and aggregation websites, *e.g*. discussion on *Triple gear* at Metafilter.

Segerman and Schleimer presented *One-half 120 cell* [4] at the Bridges Art and Mathematics conference 2012 where it won "Best Use of Mathematics", one of four awards out of 110 submissions [15]. *Triple gear*, presented at Bridges 2013, was voted "Most innovative" of 136 submissions. There have been media articles in Scientific American (reprinted in Nature), Smithsonian and Gizmodo [16].

The sculptures also featured in the 2013 Edinburgh International Science Festival, while in 2012, five large versions of the sculptures were installed at the Department of Mathematics at the University of Melbourne [17].

The breadth of the examples above demonstrates the public appetite for, and impact of, this kind of mathematical art. More than ten years after it was first published the work by Series continues to inspire, while the much more recent productions by Schleimer are rapidly attracting wide attention.

5. Sources to corroborate the impact

6. [text removed for publication]

7. Letter from internationally known computer graphics artist whose website, exhibits and exhibition posters show examples of his work inspired by Indra's Pearls.

www.josleys.com/show_gallery.php?galid=318; and his exhibits and exhibition posters: www.josleys.com/references.php

8. Letter from American software developer who confirms [3] "extremely useful in his software development and artistic endeavours" which he makes available on: www.fractalsciencekit.com/index.htm; http://fineartamerica.com/art/all/kleinian+group+fractal/all; www.redbubble.com/shop/steiner+chain+kleinian+circle+inversion; www.fractalsciencekit.com/gallery/kleingp.htm; www.fractalsciencekit.com/fractals/large/Fractal-Circle-Orbit-Trap-10.jpg

9. Website of IT professional and software developer: www.alunw.freeuk.com/oldmonths.html and http://alunw.deviantart.com/gallery/ (has also provided letter)
American programmer (whose letter corroborates use of [3]) sells through websites: www.bugman123.com/Fractals/index.html and www.zazzle.com/kleinian_double_1_15_cusp_group_poster-228240989450813261
Canadian software designer who has used [3] and posts video clips on www.brainjam.ca/tessellations.htm and www.vimeo.com/album/82895
American who incorporated algorithms into his program Swirlique on his Mystic Fractal website www.mysticfractal.com/swirlique_gallery/index.htm
with further examples on DeviantArt Gallery: www.giovannigabrieli.deviantart.com/art/Kleinian-Towers-193157543; www.riverfox1.deviantart.com/art/Mobius-Dragon-Iteration-349270804 http://theli-at.deviantart.com/art/Kleinian-drops-192676501

10. Letter from Mathematics Professor, Technical University Munich.
See also www-m10.ma.tum.de/bin/view/MatheVital/IndrasPearls/WebHomeEn

11. Book jackets: "Synchronicity: nature and Psyche in an Interconnected Universe" by J Cambray (ISBN 978-1-60344-143-8), ©2009; "The Brahma Net Sutra – Bodhisattva Precepts Handbook" (ISBN 978-1-60236-010-5), ©2009.

12. Letter from former president of Quilters Guild of the British Isles. See also www.vam.ac.uk/microsites/quilts/detail/800/Indras_Pearls
Email from Arizona received December 2012 with pictures of the design of an engagement ring and wedding stationery.

13. Website with Schleimer-Segerman's 3D prints: www.shapeways.com/shops/henryseg
[text removed for publication]

14. Websites: www.youtube.com/user/henryseg (a) Video on One-half 120-cell: http://www.youtube.com/watch?v=MyUfAs30yZk (b) Video on Triple gear: http://www.youtube.com/watch?v=I9IBQVHFeQs
Thingiverse: www.thingiverse.com/henryseg Triple gear http://www.thingiverse.com/thing:66708
Triple helix_http://www.thingiverse.com/thing:89115 http://www.metafilter.com/127408/Triple-Gear

15. The Bridges Organisation: Art and Mathematics:
http://gallery.bridgesmathart.org/exhibitions/2012-bridges-conference/henrys
http://gallery.bridgesmathart.org/exhibitions/2013-bridges-conference/schleimer-segerman

16. Short articles: http://blogs.scientificamerican.com/observations/2012/10/31/mathematicians-at-play-3-d-puzzl/ ; http://blogs.smithsonianmag.com/artscience/2013/03/fresh-off-the-3d-printer-henry-segermans-mathematical-sculptures/ ; http://gizmodo.com/5969758/

17. Sculptures: Edinburgh International Science Festival-ASCUS
www.facebook.com/media/set/?set=a.182872068531703.1073741827.148984691920441&type=1
University of Melbourne http://www.youtube.com/watch?v=gJkvlnaDLic