Institution: Queen Mary University of London (QMUL)

Unit of Assessment: B10 Mathematical Sciences

Title of case study: Spectral theory to improve the accuracy of vibrational energy predictions in complex structures such as cars, aeroplanes and buildings

1. Summary of the impact

Designs for complex structures like cars, aeroplanes and modern buildings suffer from unpredictable vibrations that lead to anything from irritating noises to dangerous structural failures. Predicting the distribution of vibrational energy in large coupled systems is an important and challenging task of major interest to industry. Until recently there was no reliable method to predict vibrations at the important mid-to-high frequency ranges.

There is a need to gain accurate predictions of vibrations at the design stage. However, previous techniques developed in the context of Quantum Chaos are too cumbersome to be used in a fast-moving commercial design setting. Bandtlow has used his expertise to develop a novel method that computes a very close approximation to these predictions but in a reasonable time. Bandtlow's method of constructing an efficient mathematical model for spectral vibrations has informed inuTech's latest product and led to enhanced performance of automobiles and aircraft.

2. Underpinning research

Predicting the distribution of vibrational energy in large coupled systems is a challenging task. The two standard approaches to this problem, Finite Element Methods (FEM) and Statistical Energy Analysis (SEA), are either restricted to the low frequency regime or rely on assumptions that are hard to control. One of the mathematical challenges at the heart of Queen Mary's research in this area is to determine high-quality numerical approximations to transfer operators, mathematical objects that play a central role in the modern theory of dynamical systems. This is an area where Bandtlow has extensive research experience.

Operator Theory is concerned with linear transformations on infinite-dimensional linear spaces. Our research is concerned with the behaviour of a linear operator on a Banach space (a vector space) under perturbations. For example, given a linear operator, our research is addressing how spectral data of the operator (its spectrum, its isolated eigenvalues and corresponding eigenvectors) can be approximated in a consistent way. In practice, spectral data of the transfer operator is difficult to calculate. Part of Bandtlow's research in this area has been concerned with methods for the computation of high quality approximations to the true spectral data and their application to concrete problems. One such method, which is particularly suitable for piecewise analytic Markov transformations, is the so-called finite section method. The underlying idea of this method [4] is to approximate the transfer operator by a sequence of finite rank operators.

The eigendata of the finite-rank approximants is readily computable and is usually taken to provide increasingly accurate approximations to the true eigendata. While a number of authors have recently used this procedure in concrete cases observing a rapid and empirically stable approximation, no proof of the convergence of this method has appeared in the literature so far.

Bandtlow's research in this area was established in 2002 while employed as a research assistant in the School of Mathematical Sciences at Queen Mary under the EPSRC Fast Stream Grant [5]. Bandtlow and Jenkinson [3,4] proved that under general conditions, the finite section method converges, and that the speed of convergence is exponential (or at least stretched exponential, depending on the dimension of the underlying system); moreover, using abstract perturbation results [1,2], rigorous and explicitly computable error-bounds for the approximation have been derived. These results have been applied to obtain convergence and error estimates for the spectral approximation of transfer operators for the Gauss map, justifying empirical results previously obtained by other authors.

3. References to the research

1. OF Bandtlow (2004). 'Estimates for norms of resolvents and an application to the perturbation of spectra'. *Math. Nachr.* 267, 3–11.

2. OF Bandtlow (2008). 'Resolvent estimates for operators belonging to exponential classes'. *Integr. Equ. Oper. Theory* 61 (2008) 21–43.

3. OF Bandtlow and Oliver Jenkinson (2008). 'On the Ruelle eigenvalue sequence'. *Ergod. Th. & Dynam. Sys.* 28 1701–1711.

4. OF Bandtlow and O Jenkinson (2008) 'Explicit eigenvalue estimates for transfer operators acting on spaces of holomorphic functions'. *Adv. Math.* 218 902–925.

5. O. Jenkinson (2002) EPSRC Fast Stream Grant GR/R64650/01 "*Approximating Spectral Data of Ruelle Transfer Operators*".

4. Details of the impact

Predicting the distribution of vibrational energy in large coupled systems is an important and challenging task of major interest to industry. For example, cars contain many moving components that may produce resonances resulting in unwanted rattles, vibrations and possibly dangerous structural weaknesses. Avoiding or eliminating these resonances is a major design task and requires a detailed knowledge of the vibrational spectrum of the coachwork and chassis.

There are two standard approaches to the problem – Finite Element Methods (FEM) and Statistical Energy Analysis (SEA) – however, these approaches are either restricted to the low frequency regime or rely on assumptions that are hard to control. In 2011, Bandtlow and Tanner (University of Nottingham) began a collaboration with inuTech (Innovative Numerical Technologies) to overcome these limitations and contribute to a black-box implementation of this novel approach.

While for low frequencies FEM is preferable, and for high frequencies SEA is the method of choice, the novel scheme really demonstrates its value and works best in the mid-frequency area, where it outperforms FEM and SEA. The method uses finite element meshes based on the actual topology of the problem, and does not need any information about eigenmodes.

Based in Nuremberg, inuTech [6] is an SME targeting engineering challenges that require application of mathematical solutions. Within inuTech there is a team of innovative and highly qualified employees, possessing many years of research and development experience, as well as profound scientific competence. InuTech has 21 employees with revenue of 1.65 Million Euros. As part of their work they develop products aimed at vibration prediction and have used Bandtlow's research and methods [1,2,3,4] to develop these products. In 2011, inuTech engaged with Bandtlow specifically as a result of his research around operator theory, with a view towards the prediction of the distribution of vibrational energy in large coupled systems. The automotive industry is an example beneficiary of inuTech's products – cars contain many moving components that may produce resonances resulting in unwanted rattles, vibrations and possibly dangerous structural weaknesses. The application of Bandltow's research and novel methods ensure that resonances are minimised and support successful design.

Between January to March 2011, Bandtlow was partly based at inuTech in Nuremberg to collaborate with inuTech, contributing to a black-box implementation of their approach. The software has progressed as a result of the application of Bandtlow's research. The algorithms developed are now stable and are able to cope with increasingly complex situations. In March 2011, the decision was taken to incorporate the algorithms into the software package. The improvements made to the software considerably speeds up and improves the accuracy of this part of the design process, resulting in cheaper and safer cars. InuTech acts as a consultant and software provider for other companies, such as Airbus, opening up possibilities for further application. In addition to EADS (Airbus), commercially valuable contacts to Range Rover, Hapag-Lloyd, and Bombardier have been established and interest in the software has been expressed by those parties. So far, the software and the consultancy expertise has been used to approach problems posed by Range Rover on shock-absorber towers of cars, by Bombardier for train design, by EADS for the Airbus fuselage, and by the German Lloyd for the design of ship hulls.

The founder and CEO [6] of inuTech says: "*The expertise offered by Bandtlow plays an essential role for our project to predict vibrational energy distributions in large built-up structures. The economic success of the implementation of these ideas depends crucially on his input and I am looking forward to continue this fruitful and beneficial collaboration on related projects*". Impact and collaboration have been put on a sustainable basis by regular meetings and visits, the latest in February 2013, which focused on considerable improvements of the software implementation as a result of the use of refined spectral approximations for optimal chassis design.

5. Sources to corroborate the impact

6. CEO and founder, inuTech (www.inuTech.de) [impact of the research for the software development (mainly concerning products for structural engineering) at inuTech and the benefit for the company in economic (financial) terms].