Institution: Aberystwyth University

Unit of Assessment: 10 (Mathematical Sciences)

Title of case study: Optimal geometry of soap films

1. Summary of the impact (indicative maximum 100 words)

Research on the optimal arrangements of soap bubbles and soap films has been used as a vehicle for public engagement in mathematics. Presentations and demonstrations have been given in both Welsh and English at various events. These have had an impact on the awareness and interest of school children in geometry and mathematics.

2. Underpinning research (indicative maximum 500 words)

Soap films minimize surface energy, which is directly proportional to surface area (or perimeter, in two dimensions), if the surface tension is constant, as is usually the case. The usefulness of wire frames dipped in soap solution to demonstrate fundamental concepts in minimization and geometry, building on Plateau's work in the 19th century, continues to this day, acting as a spur to both scientific endeavour and public engagement. It is of particular relevance to the development of mathematical models of the structure and dynamics of aqueous foams, which consist of collections of soap films, of relevance to industries include ore separation and oil recovery.

In a wire frame in the shape of a triangular prism, there is a transition between different film structures as the length of the prism changes. The transition is hysteretic, occurring at different lengths in extension and compression, and Cox and co-authors were able to show that the transition occurs “pre-emptively”, that is, before energetic arguments suggest that it should [3.1].

Mathematics, and in particular Plateau's rules, is also an important feature of calculations that explore the energy landscape of perimeter-minimizing clusters of bubbles. Similar to problems of shortest distance (cf. Steiner networks, or the Travelling Salesman problem), soap bubbles give rise to the following question: given *N* bubbles of given areas, which arrangement has least perimeter, and hence energy. This is a *packing problem*, the solution to which can indicate how best (in the sense of least interface, and possibly least deformation) to fit deformable objects within some given boundary. Ongoing effort in Aberystwyth seeks to provide candidate solutions to this problem [3.2], and to explore the conjectured solutions in the large *N* limit [3.3].

The elementary geometrical rules can even be used as the basis of predictions of the rheology of foams consisting of many bubbles. More usually, the Aberystwyth group uses energy minimization to simulate the structure, the static rheology (e.g. calculations of shear modulus [3.4]), and the slow (quasi-static) flow of foams [3.5], for example in their EPSRC/P&G-funded work [3.6].

3. References to the research (indicative maximum of six references)

[3.1] S. Hutzler, D. Weaire, S.J. Cox, A. Van der Net, and E. Janiaud (2007) Pre-empting Plateau: the nature of topological transitions in foam. Europhys. Lett. 77: 28002.
DOI: 10.1209/0295-5075/77/28002

[3.2] S.J. Cox and E. Flikkema (2010) The minimal perimeter for *N* confined deformable bubbles of equal area. Elec. J. Combinatorics 17:R45.
www.combinatorics.org/Volume_17/v17i1toc.html

[3.3] S.J. Cox, F. Morgan and F. Graner (2013) Are large perimeter-minimizing two-dimensional clusters of equal-area bubbles hexagonal or circular? Proc. Roy. Soc A 469: 20120392.
DOI: 10.1098/rspa.2012.0392
REF2 submitted.

[3.4] S.J Cox, and E.L Whittick (2006) Shear modulus of two-dimensional foams: The effect of area dispersity and disorder. Euro. Phys. J. E 21:49-56.
DOI: 10.1140/epje/i2006-10044-x

[3.5] I.T. Davies and S.J. Cox (2010) Sedimentation of an elliptical object in a two-dimensional foam. J. Non-Newt. Fl. Mech. 165:793-799.
DOI: 10.1016/j.jnnfm.2010.04.005

[3.6] EPSRC Strategic Partnership (P&G), EP/F000049/1 Characterisation, Modification and Mathematical Modelling of Sudsing, 2007-2011, £189,788.

4. Details of the impact (indicative maximum 750 words)

Aberystwyth researchers Cox and Davies have developed a number of talks and demonstrations, aimed mainly at children of secondary school age, that describe soap bubble geometry and the mathematics of minimization. These have been given under various names, including “Show me the (shortest) way to go home”, “Mathematics of Soap Bubbles” and “Bubble Magic”. The material is based upon

(i) demonstrations of Plateau's laws in wire frames, including the hysteretic transition between different minima;
(ii) seeking least area arrangements of soap films, comparing the conjectured solutions in [3] with children's intuition;
(iii) a discussion of soap-film solutions of Steiner-like problems in the plane (for example the shortest road network joining 3 cities, 4 cities, and 5 towns/cities in Wales) and local minima in complicated energy landscapes. Here numerical solutions are used to indicate the relevance of constraints (mountains!).

Interaction with participants is increased by offering the opportunity for attendees to do experiments themselves, and by running a quiz on Plateau's laws, with prizes of book tokens.

Exemplars of this activity include:

- Cox gave an invited evening talk for the Lancashire and North-west Branch of the Institute of Mathematics and its Applications and the Lancashire and Cumbria Branch of the Institute of Physics, hosted by the University of Central Lancashire, in December 2010. The audience was a mixture of academics and members of the public, including A-level pupils [5.1].

- Cox brought Frank Morgan (Williams College, US) to the UK to give a talk “Soap Bubbles and Mathematics” associated with an ICMS workshop [5.2], and publicised on his Huffington post Blog [5.3]. The event was held at Dynamic Earth in Edinburgh, attended by about sixty 14-15 year olds [5.4]. He used Cox's research results [3.2] in the talk. The children also tried out experiments for themselves (from the IMA's Big Box). A report and photograph were used as an exemplar of ICMS outreach in the LMS newsletter [5.5], and feedback from teachers and pupils included “the speaker was excellent; enthusiastic, sense of humour, involved the audience” and “Showing the applications of maths is incredibly useful” [5.6].

- Davies talks and gives demonstrations at the annual Welsh “Eisteddfod”s, both for young people (the Urdd Eisteddfod) and adults (the National Eisteddfod), which attract over 10,000 people every year. In 2012 his work was publicised by amgylchedd.com [5.7] and, in May 2013, 55 children

completed a questionnaire which indicated their increased grasp of the material.

- School visits to Aberystwyth to hear soap film talks have covered much of the country, including Llanfyllin (2009) and Bro Ddyfi, Machynlleth (2013). The feedback from Bro Ddyfi included the following comment on Twitter: “Thanks for having us, the presentations were great, very interesting" [5.8] and an email from a teacher: "The pupils enjoyed the experience, and it has certainly enriched their understanding of Mathematics in higher education” [5.9].

5. Sources to corroborate the impact (indicative maximum of 10 references)

[5.1] http://www.ima.org.uk/_db/_documents/lancs_nw_8_dec_2010.pdf

[5.2] http://icms.org.uk/workshops/soapbubble

[5.3] http://www.huffingtonpost.com/frank-morgan/international-centre-for-mathematical-sciences_b_1368994.html

[5.4] http://sites.williams.edu/Morgan/2012/03/21/icms-isoperimetric-problems-19-23-march-2012-edinburgh/

[5.5] LMS Newsletter No. 415 June 2012, pg 7. http://newsletter.lms.ac.uk/415/415_issue.pdf

[5.6] Email from the Knowledge Transfer Officer at ICMS.

[5.7] http://www.amgylchedd.com/2012/07/y-coleg-cymraeg-cenedlaethol-yn-eisteddfod-yr-urdd/ (no longer active). [Welsh]

[5.8] https://twitter.com/tudurdavies [Welsh]

[5.9] Email from the numeracy coordinator at Ysgol Bro Ddyfi.